

# TECHNIQUES FOR VISUALIZING UNCERTAINTY AND PERCEPTIONS OF RISK

## ABSTRACT

We describe risk imaging technology that decomposes risk into its two basic elements: (i) the frequency of each kind of harm associated with a hazard and (ii) the adversity of each of those harms. Because different kinds of harm are often measured along incompatible dimensions, we quantify adversity on an ordinal scale. Frequency is quantified on a ratio scale. In most risk analyses, considerable uncertainty exists regarding the actual magnitude of both frequency and adversity. For instance, sampling error, measurement error, and bias all contribute to uncertainty about frequency so differences in opinion, measurement error, and choice of dimensions lead to uncertainty about adversity. We image risk as an area circumscribed by uncertainty bounds around all of the harms. We refer to this area as the *risk profile* of a hazard. Different individuals and groups respond to uncertainty and risk differently, and the risk profile can be further focused to visualize particular risk perceptions. To do this, we specify values for *attitude parameters*. These attitudes include the overall importance of uncertainty, the meaning of disagreements between measurements or opinions, and the meaning of absence of evidence. Different values specified for these attitude parameters result in different visualizations of risk as perceived by an individual or interest group. These alternate risk visualizations may be contrasted and compared across management choices or across different risk perceivers to facilitate communication and decision-making. To illustrate the method, we image published clinical trial data.

Keywords: risk imaging, risk visualization, risk communication, risk perception, uncertainty

## 1. INTRODUCTION

Displayed visually, otherwise incomprehensible information can be rendered a powerful tool for the generation of insight. In the 100 years since Wilhelm Roentgen first used x-rays to see inside the human body, medical imaging methods such as nuclear medicine, ultrasound, computed tomography, and magnetic resonance imaging have revolutionized medicine. An x-ray, is however, a direct impression from an analog signal. Revolutions in medical imaging technologies have led to an explosion of digital data. While the scanning modalities that generate this data have received most of the credit, the megabytes of data generated by a scan would be useless without the equally ingenious techniques developed to create images of the information. This in turn enables complex information, such as the display of the chemical composition of atheroma plaques from nuclear magnetic resonance techniques, to be fully exploited by the human brain. What if risk analysts, regulators, policy decision makers, interest groups, and the general public could benefit from analogous imaging technology? Visual display systems would allow them to explore the uncertainty, differing perceptions, and disagreements that cloud risk assessment data. This article presents an imaging method that provides visualizations of risk in the face of uncertainty regarding the frequency of adverse events and of uncertainty regarding their severity.

We agree with those who believe that effective risk management requires reliable data generated by normative science <sup>(e.g. 1-3)</sup>. To make policy decisions, the response of individuals and interest groups to different risks must be considered. However, risk perception is variable, complex, often seemingly idiosyncratic and, therefore, difficult to understand or predict. This is why psychometric and socio-cultural theories of risk perception emphasize the disparity between risk

assessments by experts and by lay people<sup>(4-9)</sup>. Expert risk assessments often focus on the frequency (or probability) of an adverse event of measured magnitude, while lay assessments are conditioned by additional qualities of the hazard and characteristics of the perceiver.

The risk imaging method we propose decomposes risk into the frequency and adversity of multiple harms associated with a hazard. Quantitative uncertainty is incorporated into the calculation of these values and an image is produced of the resulting risk estimate. This image is an area circumscribed by uncertainty bounds, which characterizes risk as perceived by all individuals or groups whose contrasting risk perceptions contribute to the uncertainty. The final visualization of perceived risk depends on the attitudes of the risk perceiver towards uncertainty in frequency and uncertainty in adversity. The attitudes of particular perceivers towards uncertainty are necessary to recover and visualize their perception of risk from the bounded image of risk that pertains to all perceivers. Although intimately related, these attitudes are different from the attitudes towards risk that are the focus of most risk perception and risk communication research. The values of these attitudes may be specified by the perceiver or derived from risk perception or preference studies.

In the section that follows, we briefly review the risk perception research literature to highlight some of the important findings and concepts that have influenced our risk imaging method.

Although we have been guided by theory developed in risk perception and risk communication research, the method is theoretically eclectic and meant to be adaptable to a wide range of applications and levels of analysis. The commonality we expect across applications is the need of risk analysts to convert highly uncertain measurements of the frequency and adversity of

multiple harms associated with a potential hazard into an image of risk as variably perceived by different individuals and interest groups. Risk perception, as conceived in this sense, is unlike an x-ray image in that there is no single correct perception to be photographed. Occupational, environmental, and health risks are experienced and perceived in the context of culturally complex and highly politicized arenas<sup>(10)</sup>. An analysis informed and colored by these complexities is required<sup>(11-13)</sup>

In this article we image the risk associated with several safe and commonly used hypertension drugs. These drugs were chosen because clinical trial data was publicly available and easily accessible. The analyses presented are intended to illustrate the risk imaging method only and should not be construed as a risk analysis. Analysis of both the risk of adverse effects and the benefits received for taking the risk are necessary to understand the acceptability or tolerability of any particular risk<sup>(cf. 2, 13)</sup>. We consider only risk profiles of the adverse effects reported for these drugs.

## **1.1 Risk Perception Research**

While contrasting in many ways, the work reported here has been guided by results of the psychometric paradigm of risk perception research and the Carnegie Mellon school of risk communication research. The findings of the psychometric school have been well summarized (e.g. 5, 8, 9, 11) and are becoming widely known in the risk research community. Expert risk assessments and lay assessments differ, and lay assessments are sensitive to particular qualities of hazards. These qualities include control, voluntariness, catastrophic potential, equity, dread, newness, and the degree to which the event is unknown, among others<sup>(4, 6-8)</sup>. Risk perception researchers have proposed that the perception of risk may vary socioculturally and that different

individuals and groups may employ different mental models of risk, leading to different risk assessments<sup>(3, 14-16)</sup>; (but see Sjoberg<sup>(17)</sup> for important caveats). These models may take only partially overlapping sets of parameters as inputs, may differ structurally, and may differ regarding the definition of risk.

The Carnegie Mellon risk communication approach is presented in Morgan et al.<sup>(3)</sup> (see also<sup>(1, 18-21)</sup>). This approach builds on the mental models metaphor employed by risk perception researchers and seeks to explicitly map and contrast the mental models of expert and lay risk assessors. Much of the variability in risk perception is seen as deriving directly from variability in lay mental models. Because the goal of the approach is risk communication that informs and influences decisions and motivates actions, methods have been developed to reduce this variability, particularly through the reduction of lay misconceptions. The method also characterizes irreducible uncertainty in the form of real differences between individuals and interest groups in sociocultural context to aid decision making and clarify policy debate.

Research in risk perception and risk communication has led to a wider appreciation and respect for lay and cross-cultural risk assessments and for the pervasive influence of variability and uncertainty in all attempts to calculate and compare risks. The risk imaging and visualization methodology we describe is complementary to and builds on these general findings. It employs quantitative uncertainty techniques to calculate and visualize perceived risk in the face of uncertainty. These visualizations are intended to display risk as perceived by particular interest groups and differentiate and display perceptions arising from different mental models of risk.

These displays may then be used to describe and predict the results of alternate risk management decisions and to inform risk communication efforts.

## **2. AN IMAGE OF RISK**

In a clinical drug trial, patients are given either the drug or a placebo. Side effects are reported and their frequencies tallied. This sort of risk assessment is shown in Table I for the hypertension drug benazepril<sup>1</sup> as reported in the publicly available *Physician's Desk Reference*<sup>(22)</sup>. Although uncertainty pervades this data, no uncertainty is presented in the table. The adverse effects are neither quantitatively nor qualitatively ranked, and no uncertainty is reported regarding the estimates of frequency. Given this data in tabular (digital) form, users must infer risk by supplying their own adversity ranking and attitudes towards uncertainty in both frequency and adversity to form a risk perception. These perceptions will necessarily vary across perceivers. For instance, if a group of experts were to assign quantitative or qualitative severities to each adverse effect, opinions would certainly differ. Likewise, risk assessors who are cognizant of the importance of sample size to generalization or who exhibit less trust in the research design will perceive the reported frequencies differently than those who believe the single point estimates are likely to be representative.

It is often said that a picture is worth a thousand words, so perhaps an image of the digital data presented in Table I would aid risk assessors in forming more consistent perceptions. Figure 1 shows the data as a scatter plot. Indeed, the relative differences between the drug and the placebo for each adverse effect are easier to grasp when the data is imaged. However, this simple image suffers from many of the same shortcomings as the tabular data. For instance, the adversity axis in the figure is categorical and given in no particular order. The probability axis,

although ratio scaled, gives the data as certain points. Each risk assessor is still left with the task of supplying both an adversity scaling and attitudes regarding uncertainty before a perception can be formed. Without accounting for uncertainty in both frequency and adversity, the range within which perceptions may vary cannot be delineated. The first step we propose towards generating useful visualizations of risk as perceived requires the decomposition of risk into its adversity and frequency components.

## **2.1 Uncertainty in Adversity**

The approach ranks adversity on an ordinal scale and incorporates uncertainty as intervals that bound all individual perceptions of each adverse event. Uncertainty in perceived adversity may be due to differences across adverse events in the degree to which their harms are well known. Events with knowable adversities are those with limited, local, and well enumerated consequences. Events with many possible harms and with less well defined harms are more uncertain. Uncertainty in perceived adversity also may be due to different preferences for the choice of scales (e.g. economic cost, lives, etc. <sup>(8)</sup>), as well as sociocultural differences between individuals and groups in access to information and in the subsequent mental models they construct.

We conducted an ad hoc survey for the purposes of quantifying the severity of the adverse effects of several hypertension drugs currently on the market, including benazepril. The survey was intended to provide a snapshot of the adversity perceptions of a hypothetical population. This intent contrasts with the intent of risk communication researchers <sup>(e.g. 3)</sup> because it is the perception of *adversity* we are after, not the perception of risk, which is a function of both frequency and adversity. The sample consists of a non-randomly selected group of colleagues

and acquaintances and was collected solely to illustrate the mechanics of the risk-imaging method. The survey instrument was informal and simply asked respondents to rate, on a seven-point scale from least to most, the severity of the listed adverse effects in the context of a patient receiving a blood pressure medication. Thirty seven surveys were sent out via e-mail and 14 were returned, giving a response rate of about 38 percent.

Table II shows the results of the survey for the adverse effects associated with benazepril. Each adverse effect is associated with a range of opinions regarding its level of adversity. These can be treated as uncertain numbers characterized by intervals that range from the minimum to the maximum reported severity. Numerous aggregation methods are available to express the central tendency, including traditional measures such as mean, median, and mode. Exactly how (or whether) opinions *should* be aggregated depends on the attitudes of particular perceivers regarding uncertainty, and these attitudes are expected to vary across perceivers. For example, individuals whose attitudes towards uncertainty include a high level of trust and confidence that the scope and severity of the adverse effect is well known and understood may perceive the adversity as the average or modal opinion. In contrast, those who feel that the adverse effect is poorly understood and may result in a wide range of outcomes may perceive the adversity as much more uncertain. The perceptions of these individuals may be better characterized by a range of possible adversities.

When combined with the precise frequency data in Table I, the data in Table II can be imaged in a rudimentary fashion. In Figure 2, the seven adverse reactions are shown as intervals. The intervals overlap extensively, indicating a wide range of opinions regarding the severity of each

adverse effect. A central tendency measure (arithmetic mean) is shown in the figure for each adverse effect as a small vertical black line.

## **2.2 Uncertainty in Frequency**

Frequencies are relative counts of event occurrences, often expressed as the ratio of the number of occurrences to a count of all members of a reference class at risk of occurrence. The ratio of specific events to the total number of trials defines the probability of the event. Probabilities calculated from a representative sample are estimates of the probabilities faced by the population from which the sample was drawn. All such estimates are uncertain. Adverse events whose frequencies are more difficult to determine, because for example they are rare, new, or difficult to observe, have more uncertainty associated with their risk image than those whose frequencies are more easily known. In addition, samples may be biased due to under- or over-reporting of incidence rates or other factors that skew selection. Our approach measures frequency on a ratio scale and incorporates uncertainty as intervals that bound the uncertain value. Figure 3 shows the uncertainty regarding probability of occurrence of each of benazepril's seven reported adverse effects. The minima and maxima of the intervals are 99% confidence limits<sup>2</sup>, which are a function of both frequency and sample size. A central tendency measure (the reported probability) is shown as a horizontal black line crossing each adverse effect's uncertain probability interval. The vertical lines are positioned at the mean adversity rate for each effect.

## **2.3 The Risk Profile**

The frequency and adversity of headaches, or any other single adverse effect, does not constitute a complete risk assessment for benazepril. Instead, it is the combination of adverse effects and frequencies that describe the risk. Because we have ranked the adverse effects on a scale of

severity, we may dispense with tracking individual adverse effects and instead calculate the frequency of each adversity level. We refer to this graph as a *risk profile*.

Uncertainty in adversity and in probability are graphed in Figure 4. Together they circumscribe an area that includes all of the combinations of adversity and probability that are consistent with the uncertain data. In Figure 4, dotted black lines form a box that illustrates the extent of this area for the headache adverse effect. A similar box can be drawn around each adverse effect.

Since uncertainty regarding the adversity level of individual effects can be quite high, individual effects can overlap extensively. To draw a risk profile, some method is needed to combine, on the vertical scale, the multiple uncertain adverse effect probabilities at each adversity level. In Figure 4, for example, both nausea and postural dizziness were scored at adversity level 6 by some survey respondents. What, then, is the uncertain probability that an adversity level 6 adverse effect will occur?

We use the classical Fréchet inequality<sup>(23)</sup> to bound probabilities at each adversity level where two or more adversities overlap. For example, to apply the Fréchet inequality at adversity level 6 in Figure 4, note that the probabilities of both nausea and postural dizziness are in the interval<sup>3</sup> [0.004, 0.022]. These intervals represent the 99% confidence intervals around each adverse effect's observed probability. How many individuals in a population of people who take the drug will experience at least one of the level-6 adverse effects? According to the Fréchet inequality, the minimum probability of any adversity level must be the largest minimum probability of all adverse effects at that level. In the benazepril example, the minimum

probability of level-6 adversities must be 0.004, which is the same for both adverse effects. This means that at least four in 1000 patients will experience a level-6 adverse effect, either postural dizziness or nausea or both. Imagine that another adverse effect, say headache, had also been rated as adversity level 6. The minimum probability of headache is 0.04, which is an order of magnitude higher than postural dizziness or nausea. If headache were rated as adversity level 6, at least 4 percent of patients would experience a level 6 adverse effect since at least 4 percent of patients would experience headaches.

To calculate the maximum probability that a patient will experience a level-6 adverse effect, the dependence between effects rated as level 6 must be considered. If all patients reporting nausea also reported postural dizziness, then the maximum probability of patients experiencing a level-6 effect would be the smaller of the maximums of each adverse effect. However, if no patient reporting nausea also reported postural dizziness, then the maximum probability would be the sum of the maximum probabilities for each adversity: in this case  $0.022 + 0.022 = 0.044$ .

Uncertainty bounding via the Fréchet inequality accounts for the unknown dependence structure existing between each of the listed adverse effects<sup>4</sup>. In the face of ignorance regarding the dependence between adverse effects, the sum of maximum probabilities (or one if the sum of the maxima is greater than one) is the upper bound. The uncertain probability for level-6 adversities is therefore given by the interval  $[0.004, 0.044]$ . Assuming that the original sampling was representative, some number between 4 and 44 of every 1000 patients taking benazepril will experience a level-6 adverse effect. Figure 5 shows the benazepril risk profile with full uncertainty regarding the probability of each adversity level and the severity of each adverse effect imaged.

### 3. ATTITUDES TOWARDS UNCERTAINTY

The risk of adverse effects imaged in Figure 5 expresses the uncertainty in both severity and probability present in the risk data. However, expressing and imaging the uncertainty is only the first step towards visualizing risk as it is perceived. The next step in the risk visualization is to locate the risk perceptions of individuals and interest groups within the image of uncertain risk. To accomplish this, we quantify their attitudes regarding the importance of uncertainty, the meaning of disagreements between measurements, opinions, or judgments, and the meaning of absence of evidence. Based upon the same underlying risk image, different combinations of attitude values lead to contrasting visualizations of perceived risk. These differing risk visualizations can be contrasted and compared across alternate choices or scenarios, or across different risk perceivers, to facilitate communication and decision making.

The three attitude scales used to quantify attitudes towards uncertainty are shown in Figure 6. The attitudes are labeled *Burden of proof*, *Dispute tolerance*, and *Uncertainty display*. Selecting a value for each attitude alters the visualized image of risk. The attitude variables are interdependent, in that each requires input from the others to determine its effect on the risk visualization. The definition and effect of each attitude on the risk perception visualization are discussed in the following sections.

#### 3.1 Burden of proof

The choice of an attitude value on the *Burden of proof* scale emphasizes different portions of the risk image. *Burden of proof* quantifies the perceiver's attitude towards the meaning of absence of evidence. Moving the slider to the left indicates that, in the face of little evidence of harm, an adverse event can be assumed to be safe until proven otherwise. Moving that slider to the right

indicates that the perceiver believes that when data is lacking, adverse events should be assumed to be harmful. The importance of this aspect of risk perception has been widely discussed under the rubric of the precautionary principle (e.g. 24-26). Issues such as trust in authorities and the identity of those who suffer the consequences (equity) are involved in the setting of this attitude.

With the other two sliders set to intermediate attitude preferences, Figure 7 shows the effect of moving the *Burden of proof* slider in the benazepril example. The lighter blue area in each panel shows the benazepril risk image with maximum uncertainty in both adversity and probability displayed. This is the image displayed when both *Dispute tolerance* and *Uncertainty display* are set to their rightmost (i.e. maximum uncertainty display) values. The darker blue areas represent risk visualizations at the extreme (Panels A and C) and central (Panel B) values of *Burden of proof*. All else equal, risk perceivers who believe that absence of evidence of harm is indicative of little potential likelihood of harm perceive the risk profile shown in Panel A. Those who interpret paucity of evidence under worst-case scenarios would perceive the risk profile shown in Panel C.

The *Burden of proof* slider moves the center point of each adverse effect in the risk profile. When the slider is in the middle position (Panel B in Figure 7), each center point equals the central tendency reported by the user, for example the mean adversity and the reported frequency. When the slider is moved to the right, the center point of each adverse effect moves towards the maximum possible probability and the maximum possible adversity in direct proportion to the distance the slider is moved. Moving the slider to the left moves each center point towards its respective minimum possible probability and adversity. The center points

themselves are not visualized, however they act as inputs to the visualized risk profile bounds calculated by the *Dispute tolerance* and *Uncertainty display* sliders.

### **3.2 Dispute tolerance**

The *Dispute tolerance* slider is intended to reflect the risk perceiver's interpretation of differences in opinion and judgment, differences in models, and differences in information regarding the severity of an adverse event. In the benazepril example, the adversity data consists of individual opinions regarding the severity of each of the seven adverse effects. These opinions vary widely. Respondents to the survey included lay people, professional risk assessors, medical doctors and public health experts. Some risk perceivers may believe that the best guess regarding adversity is some sort of consensus among those polled, e.g. an average or other summary statistic. Other perceivers may feel that the diversity in opinion is important itself. If the adversity is very poorly known or understood, perhaps each opinion regarding adversity is as good as any other and all should be considered equally valid. An intermediate position would be that the extreme opinions, as measured from the central tendency, may be overlooked while the diversity around the central tendency should be maintained.

In Figure 8, the effect of moving the *Dispute tolerance* slider from left (least uncertainty displayed) to right (most uncertainty displayed) is shown for intermediate settings of the other two attitudes. Moving the slider completely to the left (Panel A) requires a consensus and uncertainty due to diversity of opinions is removed with an aggregation operation (the arithmetic mean, in this example). Moving the slider completely to the right (Panel C) indicates the risk perceiver feels that every opinion deserves full and equal consideration. Moving the slider to a middle position (Panel B) suggests that extreme opinions, as judged by distance from the central

tendency, may be overlooked, but that variance in more common or average opinions is still not entirely reducible and must be accounted for.

The *Dispute tolerance* slider works by expanding or contracting the uncertainty bounds displayed relative to the central tendency and the overall perception of the importance of uncertainty. The central tendency preference is set by attitudes regarding the *Burden of proof*, as discussed above. The *Dispute tolerance* slider operates only on uncertainty present in adversity, and has no influence on the frequency axis.

### **3.3 Uncertainty display**

The *Uncertainty display* slider gauges the risk perceiver's attitudes towards the importance of uncertainty in a risk assessment. It works in a manner similar to the *Dispute tolerance* slider, however it simultaneously affects both uncertainty in adversity and uncertainty in probability. Uncertainty is most important as a factor influencing risk perception when risk is difficult to observe, not well known, or arises from a novel source. Trust in the ability of analysts and officials to calculate and control risks influences the importance of uncertainty as well. Finally, the potential scope of consequences may impact how much uncertainty is perceived. The *Uncertainty display* slider is intended to capture some of these effects.

In Figure 9, the effect of moving the slider to the left (less uncertainty displayed) is shown in Panel A. Moving it completely to the left (not shown) removes all uncertainty and expresses perceived risk as certain points at the central tendencies of adversity and probability. These point estimates of risk are akin to deterministic expert risk assessments common before the adoption of probabilistic techniques, and still sometimes required by regulatory agencies. These

central tendencies are affected by attitudes towards *Burden of proof*, but otherwise express no element of uncertainty. From this extreme attitude towards the importance of uncertainty, moving the slider to the right indicates an increasing acknowledgement of the uncertain nature of the particular risk (Panels A to B). Moving the slider to the far right (Panel C) expresses all of the uncertainty present in the measure of probability, and all of the uncertainty allowed in the measure of adversity given the risk perceiver's expressed attitudes towards *Dispute tolerance*. Expansion and contraction of the uncertainty bounds around the risk profile occur with respect to the central tendency, as adjusted by *Burden of proof*, and with respect to the bounds on uncertainty in adversity imposed by *Dispute tolerance*.

#### **4. VISUALIZING RISK PROFILES**

In this section, we use the methods described above to compute risk visualizations and compare visualized risk profiles across different perceivers and across different risks.

##### **4.1 Comparing Perceptions**

Research and data collection would be needed to accurately quantify attitudes towards uncertainty in frequency and in adversity expressed by different individuals and groups. Here, we guess what such attitudes might be for two groups based on stereotypes in order to demonstrate our approach to comparing risk perceptions.

In Figure 10 Panel A, we return to the risk image of benazepril, this time visualized from the perspective of a hypothetical government regulator. We emphasize that the attitude settings chosen are for illustrative purposes only and not based on any data. We imagine that a government regulator prefers little uncertainty, is interested in a reasonable amount of diversity

of opinion, and, given that the usual drug testing procedures were followed, expects that if no harm has been shown, the benefit of the doubt is weighted towards the supposition that no harm will occur. These settings are shown on the three attitude sliders to the right in Panel A of Figure 10.

In Panel B, we visualize the risk data from benazepril as we imagine a representative of a public interest advocacy group might perceive it. Here, uncertainty is considered very important, broad diversity of opinion is accepted, and the *Burden of proof* is weighted highly towards the precautionary end of the spectrum.

Compare the risk visualizations shown in Panel A and Panel B. Recall that both risk visualizations are based on the same uncertain data, yet different perceptions emerge due to different attitudes. These attitudes are not towards risk per se, but towards the expression of the uncertainty inherent in the two components of risk. Both visualizations would seem consistent with available data given the stated uncertainty. We expect that comparisons of risk profile visualizations such as this may be useful to analysts and decision-makers needing to understand and predict the response of individuals and interest groups.

## **4.2 Comparing Risks**

Risk perception researchers often compare risks of different hazards as perceived in order to better understand the determinants of risk perception<sup>(4, 6)</sup>. Risk analysts often use risks of well-known hazards as benchmarks to gauge the risks of new or potential hazards. In neither case are the effects of uncertainty on the estimate of risk, nor attitudes towards that uncertainty, routinely quantified or incorporated into the analysis. This risk visualization method can image risk with

quantitative uncertainty incorporated into the risk estimate, and the risks of different hazards can be compared. The actual visualizations, in keeping with the findings of risk perception research, are sensitive to the attitudes of the risk perceiver.

Figure 11 provides an example of a comparison of risks from different hazards. In the figure, risk visualizations for three hypertension drugs currently on the market, benazepril, lisinopril<sup>5</sup>, and atenolol<sup>6</sup>, are shown. Adverse reaction frequency data for all drugs were derived from package inserts reprinted in the *PDR* <sup>(22)</sup>. The severity of the adverse reactions listed in the *PDR* was derived from our *ad hoc* survey discussed above. Panel A in the figure shows the risk profile comparison as perceived by the hypothetical government regulator, and Panel B by the public interest advocate.

## 5. DISCUSSION

Although this work is preliminary and in need of further development and testing, the attitude parameters and the decomposition of risk into uncertain adversity and uncertain frequency components are consistent with both theory and data from the risk perception, communication, and decision-making under uncertainty fields. For example, in his book, *Human Judgment and Social Policy*, Hammond <sup>(26)</sup> discusses the importance of uncertainty in all aspects of decision making and risk analysis. Under the rubric of “subjective uncertainty”, he emphasizes several important attitudes towards uncertainty that impact judgment. These correspond well with those we employ. He outlines the intricacies of accounting for variation in *Dispute tolerance* with an example from the *Wall Street Journal*. The *Journal* employs forty-four economists to make a yearly economic forecast, yet only reports the average of these economists’ opinions. The uncertainty inherent in the data is ignored, and ignoring the uncertainty represents only one of

many possible perceptions of the data. In addition, Hammond's treatment of "false positives" and "false negatives" is closely akin to our *Burden of proof* attitude towards uncertainty. He links this idea to issues of social justice, fairness, and equity.

The Carnegie Mellon approach to risk communication delineates three steps towards creating effective risk communications. The first step is the creation of an expert model. The second step is the characterization of the population for which the communication is intended in terms of the mental models in use for perceiving risk. The last step is the creation of communication materials designed to reduce the misconceptions identified in the second step and provide normatively accurate information necessary to achieve accurate risk perceptions<sup>(3)</sup>. Our approach to visualizing risk perception is intended to provide a complimentary tool for achieving their second step: the visualization of risk as perceived by individuals and interest groups composing the population under study. However, our approach is graphical and does not aim to derive the explicit mental models underlying those perceptions. Our goal is to visualize the variability in these perceptions within the population as a tool to aid in risk management decisions.

In an influential paper, Slovic<sup>(6)</sup> presents an analysis of the perceived risk of 81 hazards. He labels two primary factors "unknown risk" (factor 2) and "dread" (factor 1). The characteristics of unknown risk include observability, awareness of exposure, timing from exposure to effect, how long the risk has been known, and whether the risk is known to science. We hypothesize that the characteristics of the unknown risk component are related directly to uncertainty regarding the frequency of event occurrence. Estimates of event frequency depend on reliable

counts. Low observability, delayed effects, and less time since observations began all increase uncertainty regarding the reliability of estimates of the frequency of occurrence. By incorporating and displaying uncertainty in frequency, the risk imaging method addresses this component of perceived risk.

The dread factor may relate directly to uncertainty regarding adversity. The characteristics associated with dread include controllability, catastrophic potential, fatality potential, long term harm, harm that is difficult to reduce, and whether exposure is voluntary. Many of these characteristics seem to speak to uncertainty regarding the severity of the hazard. For example, hazards that are difficult to control, may harm a large number of people, may kill, and may last a long time, are hazards whose true severity is very difficult to gauge. In addition, the *number* of ways in which a hazard can cause harm seems relevant to the dread scale. For example, in Slovic's <sup>(6)</sup> Figure 1, we see power mowers, smoking (disease), coal mining (disease), coal mining (accidents) and nuclear weapons ranking in this order from least to most on the dread scale. Uncertainty due to an increase in the number of harms that could result from each hazard may be evident on this axis. Lawn mowers can cause lacerations, smoking is related to two or three primary diseases, and coal mining to four to six diseases. Coal mining accidents, on the other hand, are associated with a long list of harms, both fatal and otherwise. Nuclear weapons have perhaps the longest list of potential harms. The severity of a hazard depends on exactly what happens when it occurs. Perceptions of risk due to harms that result in death may, for instance, hinge on whether death comes in one or in many forms. Uncertainty about severity increases as the number of potential harms increases. The risk imaging method is intended to capture and display this uncertainty.

This method for imaging and visualizing risk perception in the face of uncertainty treats all perceptions of risk within the risk image as equally valid. Individual perceptions are visualized by specifying attitudes towards uncertainty regarding the frequency and severity of adverse events. These attitudes allow for the location of individual risk perceptions because they specify how much and what kind of uncertainty is perceived. These attitudes also correspond in many ways to characteristics isolated by risk perception researchers as important to individuals' determinations of risk.

Adversity survey data is needed for a variety of hazards. A method for determining the actual attitudes towards uncertainty held by different individuals and interest groups is needed. Self-reporting of attitudes may be viable in this regard, but case studies of past decisions, opinions, or actions also may provide the necessary information. How these attitudes change across individual or classes of harms is unexplored. How these attitudes change over time within individuals or classes of harms is also unknown. Human minds may sense differences in the extent and type of uncertainty from one hazard to another as well as uncertainty in frequencies. Our work is preliminary and this tool is intended help us explore these factors and to lead to better risk informed decision making.

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Adverse Reaction	Frequency Data			
	Benazepril (N=964)		Placebo (N=496)	
	<i>n</i>	%	<i>n</i>	%
Headache	60	6.22%	21	4.23%
Dizziness	35	3.63%	12	2.42%
Fatigue	23	2.39%	11	2.22%
Somnolence	15	1.56%	2	0.40%
Postural Dizziness	14	1.45%	1	0.20%
Nausea	13	1.35%	5	1.01%
Cough	12	1.24%	5	1.01%

**Table I** Frequencies of adverse effects for benazepril. Data are as given in the *PDR*<sup>(22)</sup>.

Adverse Reaction	Adversity Data			
	<i>N</i>	min	mean	max
Headache	14	1	2.04	4
Dizziness	14	1	2.71	5
Fatigue	14	1	2.14	4
Somnolence	14	1	1.86	4
Postural Dizziness	14	1	3.07	6
Nausea	14	1	2.86	6
Cough	14	1	2.21	5

**Table II. Severity of adverse effects of benazepril as rated by respondents to an ad hoc survey. Adverse effects were scored on a scale of 1 (least severe) to 7 (most severe). *N* is the number of survey respondents, min is the minimum adversity reported, max is the maximum adversity reported, and mean is the average adversity score across all respondents.**

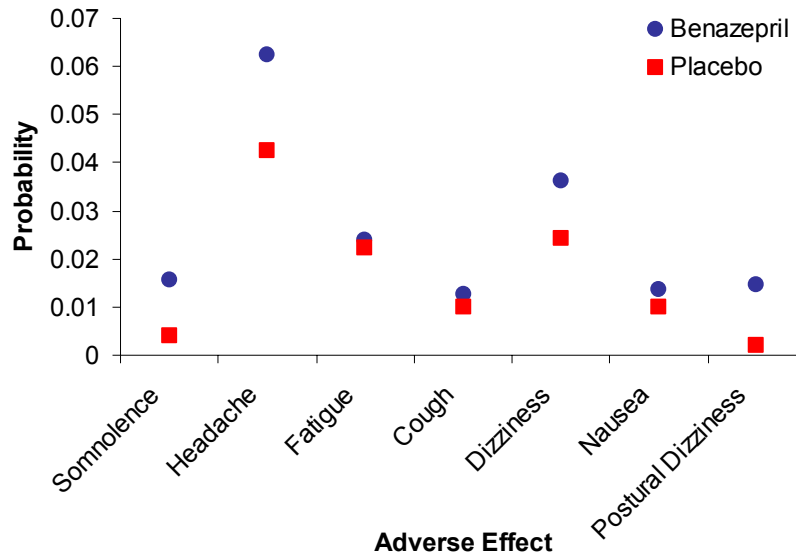
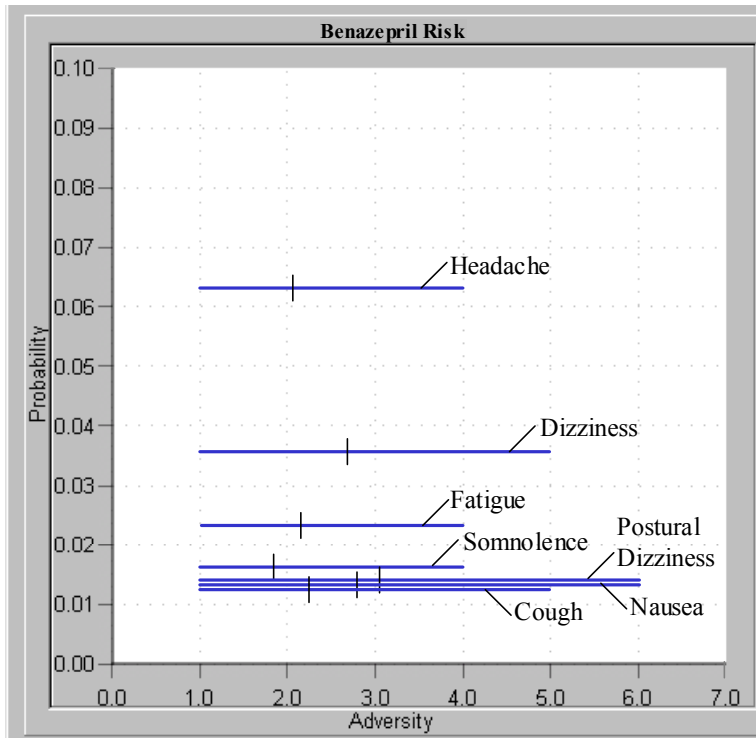
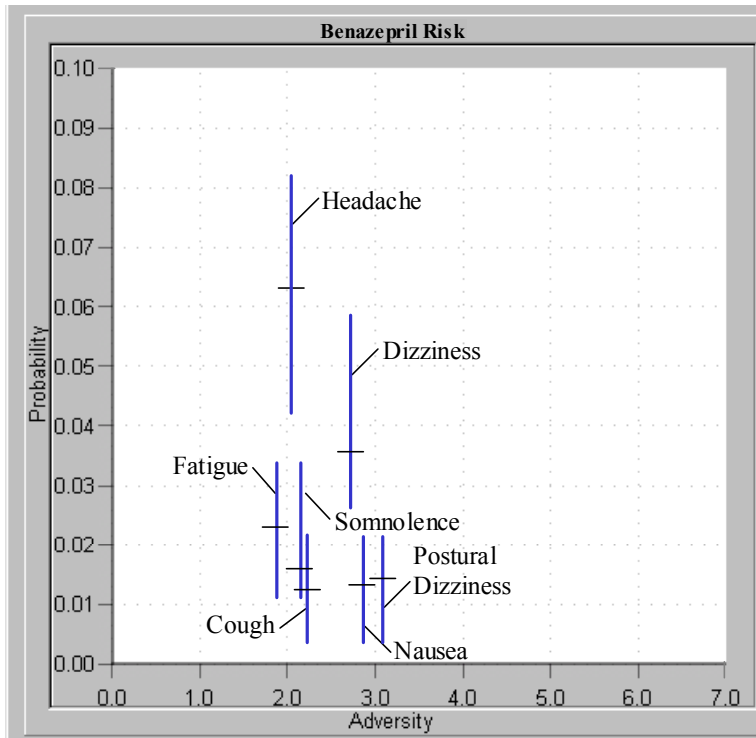


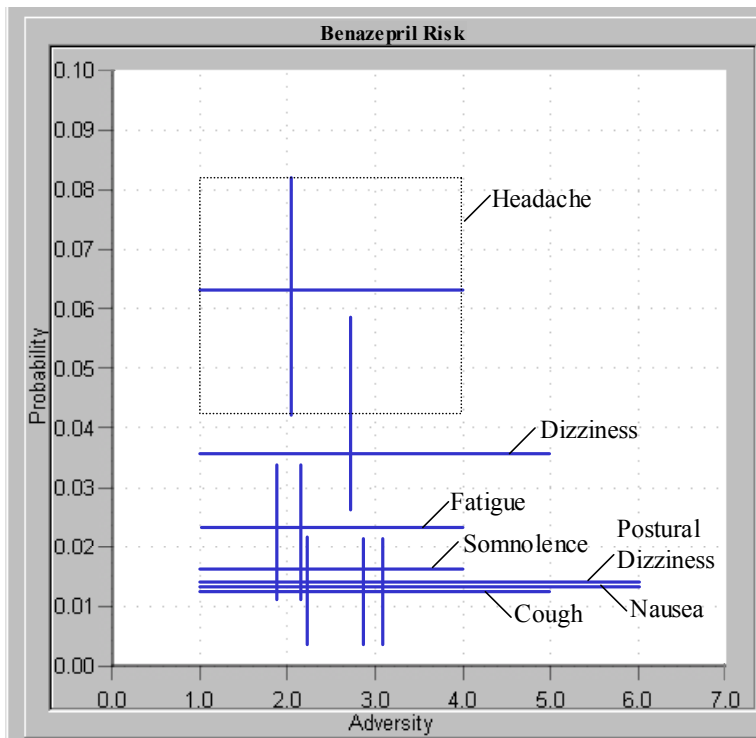
Figure 1 Frequencies of adverse effects for users of the hypertension drug benazepril. The data are from the package insert that is reprinted in the *PDR* <sup>(22)</sup>.



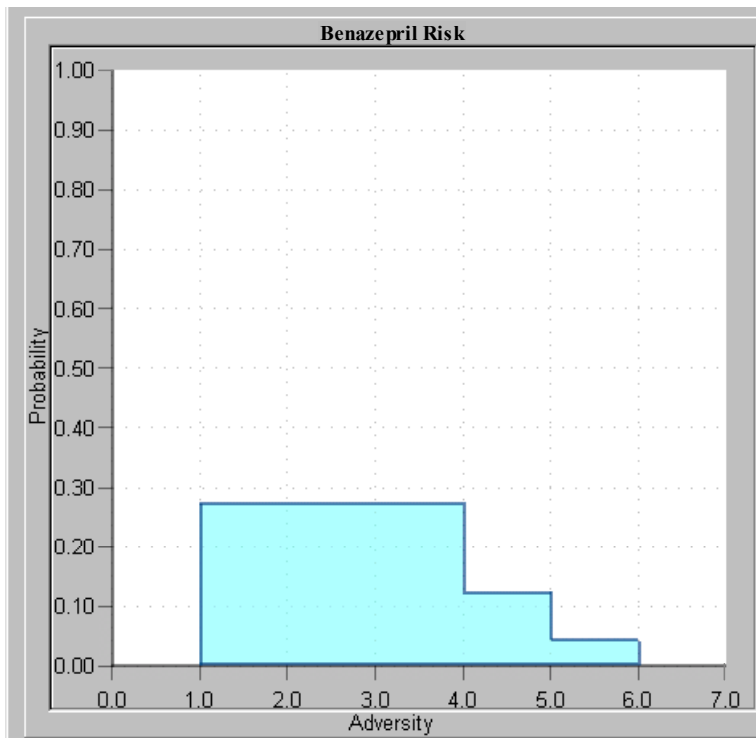
**Figure 2. Severity of seven adverse reactions to benazepril shown as uncertain numbers by precise probability. Vertical black lines indicate mean adversity rating for each adverse effect.**



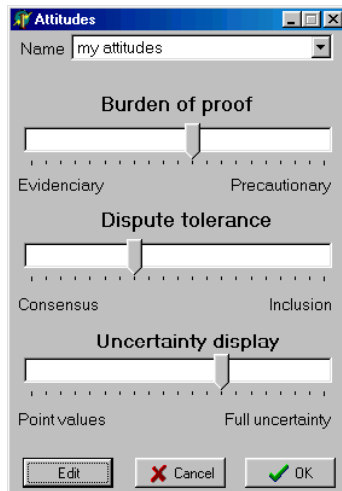
**Figure 3. Probability of seven adverse reactions to benazepril shown as uncertain numbers by average adversity rating. Horizontal black lines indicate observed probability for each adverse effect.**



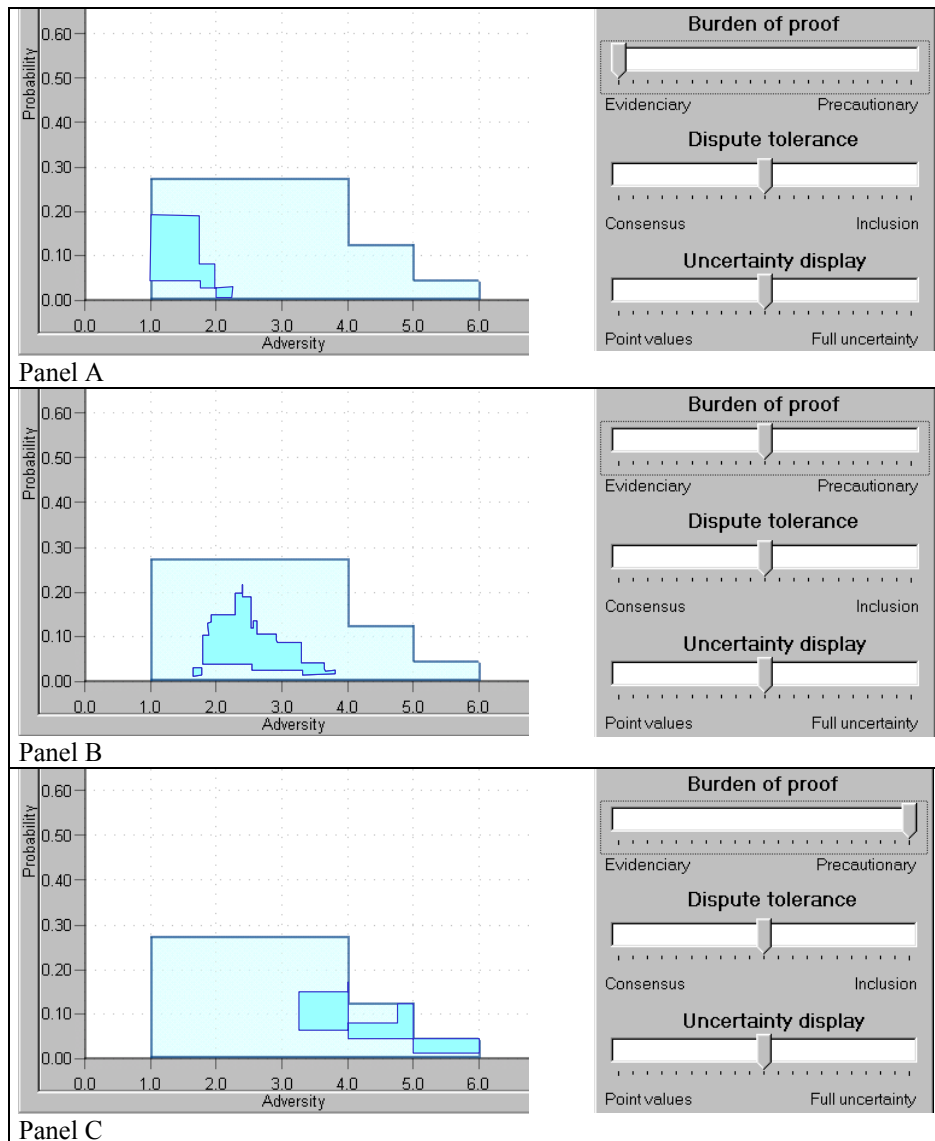
**Figure 4.** Adversity and probability of seven adverse reactions to benazepril shown as uncertain numbers. The dotted black lines forming a box show bounds around probability and adversity combinations for headache that are consistent with the uncertain data.



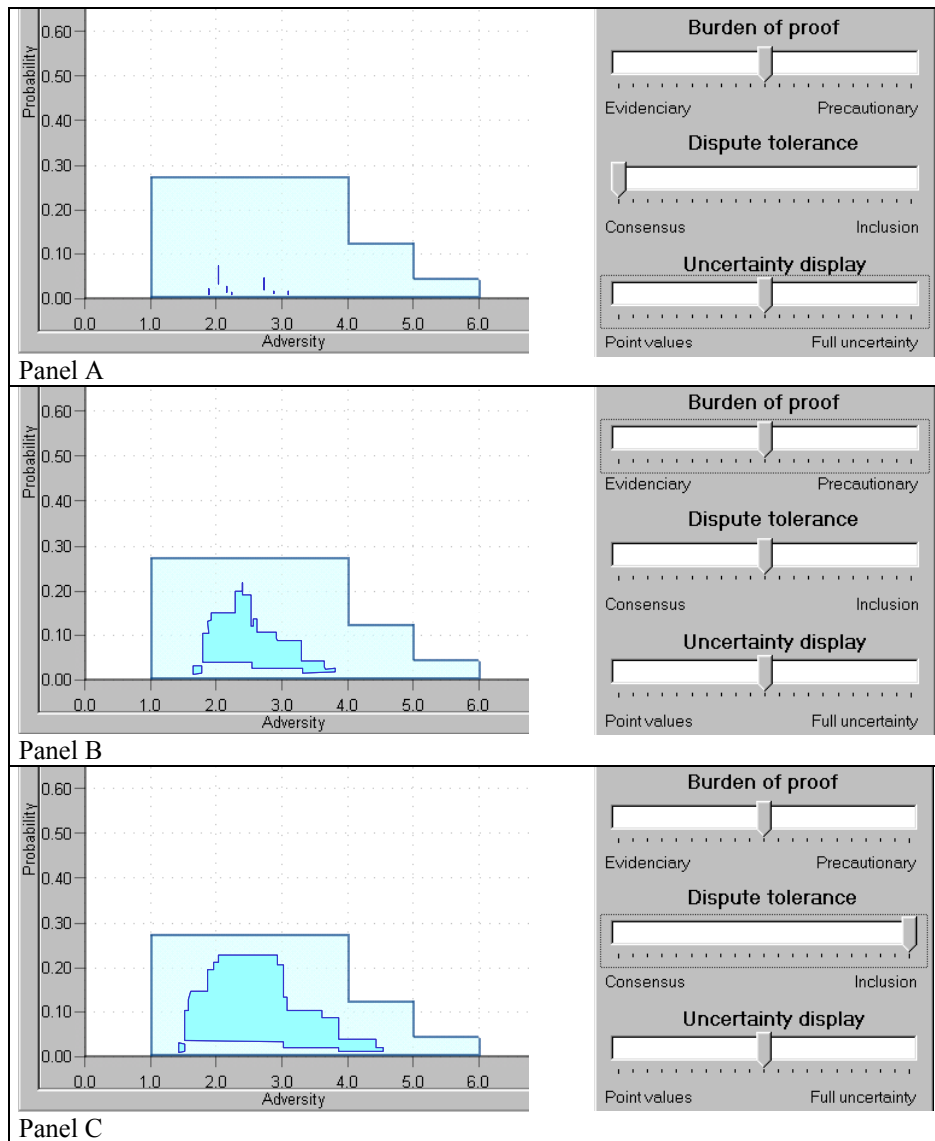
**Figure 5. Benazepril risk profile with full uncertainty imaged on both the adversity and the probability axes.**



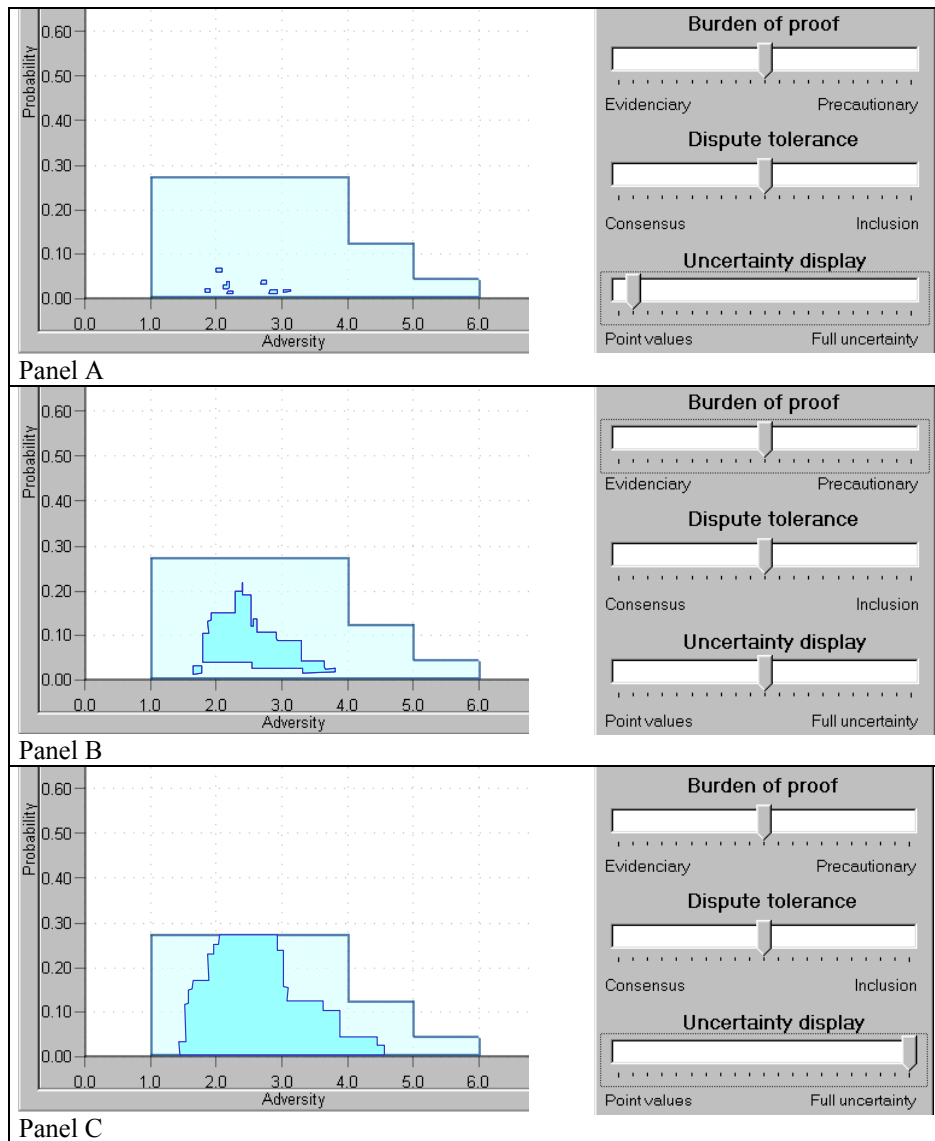
**Figure 6. Attitude quantification sliders used to specify a particular risk perceiver's attitudes towards uncertainty.**



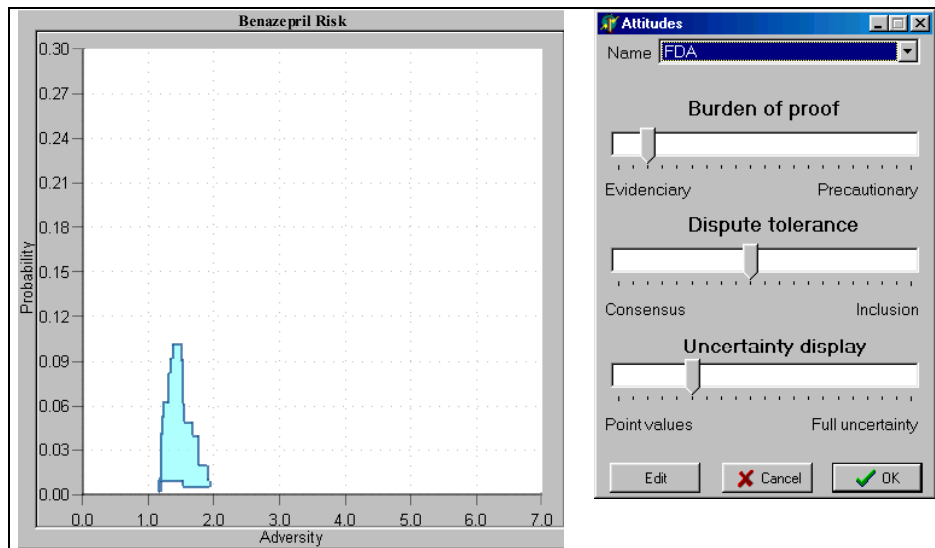
**Figure 7. The effect of the *Burden of proof* attitude slider. The darker blue profiles in each panel result when the *Dispute tolerance* and *Uncertainty display* sliders are set to intermediate values and the *Burden of proof* slider is varied.**



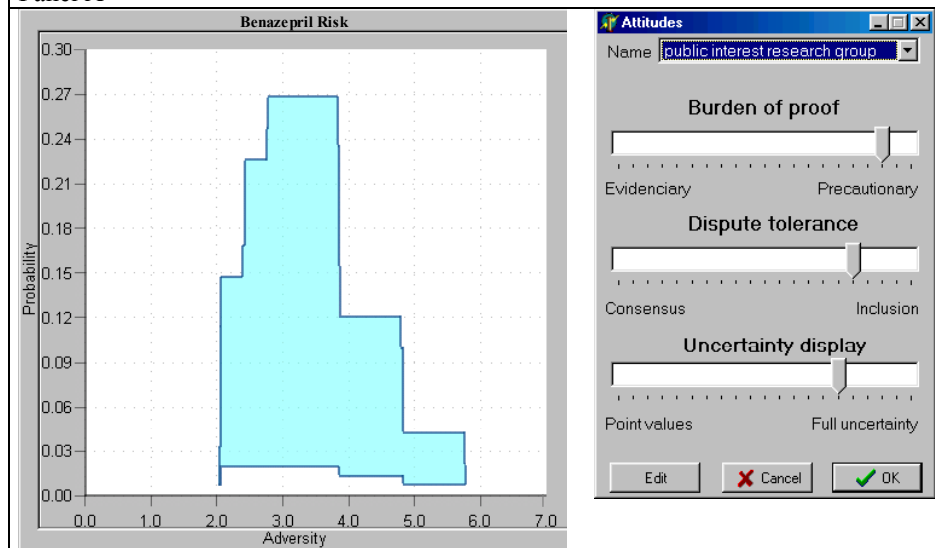
**Figure 8.** The effect of the *Dispute tolerance* attitude slider. The darker blue profiles in each panel result when the *Burden of proof* and *Uncertainty display* sliders are set to intermediate values and the *Dispute tolerance* slider is varied.



**Figure 9.** The effect of the *Uncertainty display* attitude slider. The darker blue profiles in each panel result when the *Burden of proof* and *Dispute tolerance* sliders are set to intermediate values and the *Uncertainty display* slider is varied.

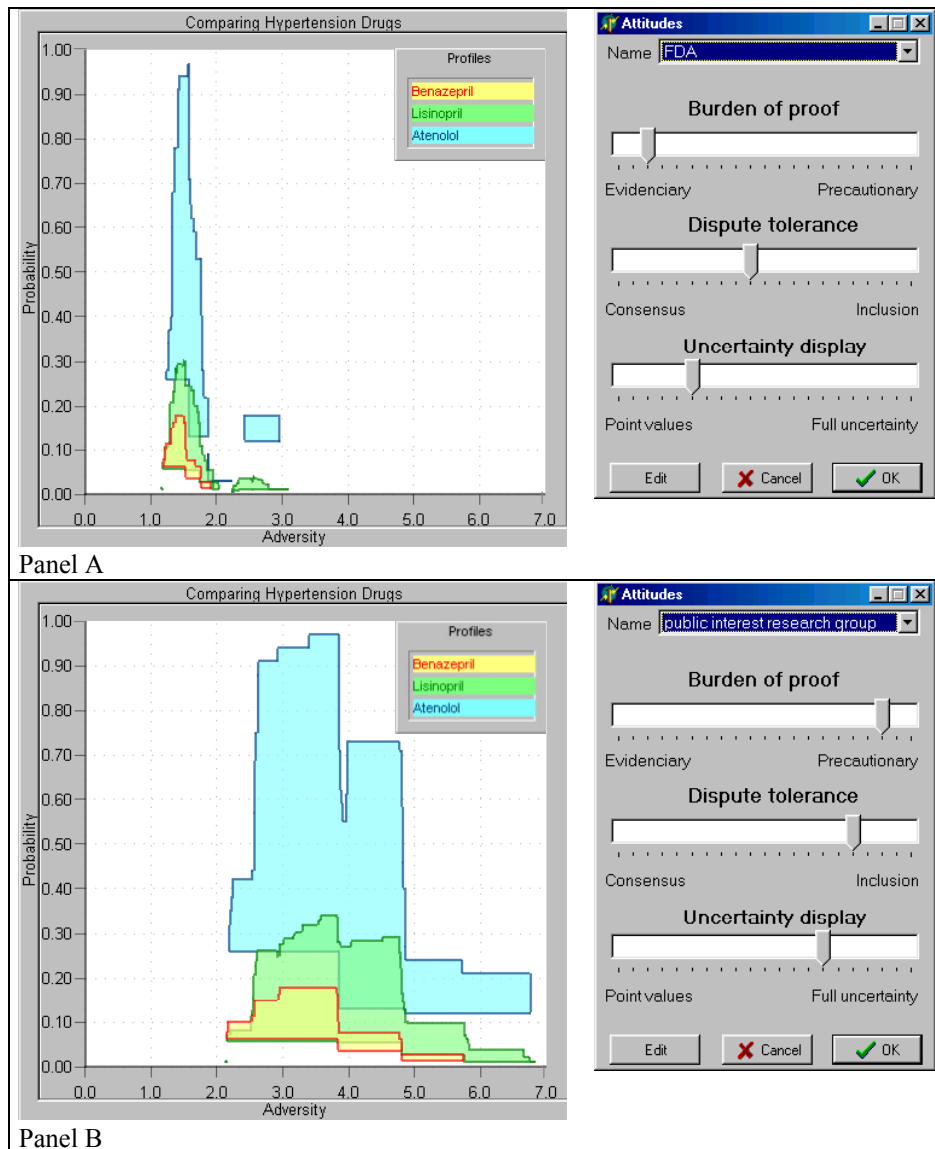


Panel A



Panel B

**Figure 10. Comparison of risk perception profiles for benazepril. Panel A shows a hypothetical government regulator’s perception of the risk of benazepril. Panel B shows a hypothetical public interest advocacy group’s perception.**



**Figure 11. Comparing three hypertension drugs from two perspectives. Frequency data from the *PDR*<sup>(22)</sup>. Adversity data from the *ad hoc* survey (see text). Panel A shows hypothetical government regulator risk perception profiles. Panel B shows hypothetical public interest advocate risk perception profiles. The benazepril risk profile is shown in yellow, lisinopril in green, and atenolol in blue.**

## NOTES

<sup>1</sup> Benazepril is the generic name for Novartis' product Lotensin, which has been off patent since 2003.

<sup>2</sup> Because an event either occurs or it does not, the distribution is binomial and confidence intervals around each frequency may be computed <sup>(27,28)</sup>. The confidence intervals are widest at small sample sizes and at intermediate frequencies. These calculations assume an unbiased sample has been obtained.

<sup>3</sup>The square bracket notation  $[x, y]$  expresses an interval where  $x$  is the minimum value and  $y$  is the maximum value of the interval.

<sup>4</sup> In some cases, clinicians may have the joint data needed to estimate the dependency structure between adverse effects. When known, this information can be incorporated into the calculation using additional probability bounding techniques, and narrower uncertainty bounds can be generated around the uncertain frequencies. The present example is the more general case where the dependency structure is entirely unknown.

<sup>5</sup> Lisinopril is the generic name for AstraZeneca's product Zestril, which is off patent.

<sup>6</sup> Atenolol is the generic name for AstraZeneca's product Tenormin, which is off patent.