

Sensitivity analysis using probability bounding

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Abstract

Probability bounds analysis (PBA) provides analysts a convenient means to characterize the neighborhood of possible results that would be obtained from plausible alternative inputs in probabilistic calculations. We show the relationship between PBA and the methods of interval analysis and probabilistic uncertainty analysis from which it is jointly derived, and indicate how the method can be used to assess the quality of probabilistic models such as those developed in Monte Carlo simulations for risk analyses. We also illustrate how a sensitivity analysis can be conducted within a PBA by pinching inputs to precise distributions or real values.

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1. Introduction

Uncertainty analysis is a systematic study in which “a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified” [1]. There are two disparate ways to effect such a study. One natural way is to bound the neighborhood with interval ranges. Another natural way is to ascribe a probability distribution to the elements in the neighborhood. Consider, for example, the context of a deterministic calculation. When the model involves uncertainty about the real-valued quantities used in the calculation, uncertainty analysis can be conducted via interval analysis [2–5]. Probability theory, implemented perhaps by Monte Carlo simulation, can also be used as an uncertainty analysis of a deterministic calculation because it yields a distribution describing the probability of alternative possible values about a point estimate [6–9]. In the figure below these two possible paths are shown as right and left downward arrows, respectively (Fig. 1).

Of course, the calculations on which it might be desirable to conduct uncertainty analyses are not all deterministic. In fact, many of them are already probabilistic, as is the case

in most modern risk analyses and safety assessments. One could construct a probabilistic uncertainty analysis of a probabilistic calculation. The resulting analysis would be a second-order probabilistic assessment. However, such studies can be difficult to conduct because of the large number of calculations that are required. It is also sometimes difficult to visualize the results in a way that is easily comprehensible. Alternatively, one could apply bounding arguments to the probabilistic calculation and arrive at interval versions of probability distributions. We call such calculations “probability bounds analysis” (PBA) [10–12]. This approach represents the uncertainty about a probability distribution by the set of cumulative distribution functions lying entirely within a pair of bounding distribution functions called a “probability box” or a “p-box”. (The mathematical definition of a p-box is given in a companion paper [13] in this journal issue.) PBA is an uncertainty analysis of a probabilistic calculation because it defines neighborhoods of probability distributions (i.e. the p-boxes) that represent the uncertainty about imperfectly known input distributions and projects this uncertainty through the model to identify a neighborhood of answers (also characterized by a p-box) in a way that guarantees the resulting bounds will entirely enclose the cumulative distribution function of the output. A probability distribution is to a p-box the way a real scalar number is to an interval. The bounding distributions of the p-box

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