

Mental Calculators that perceive risk and how we may educate them

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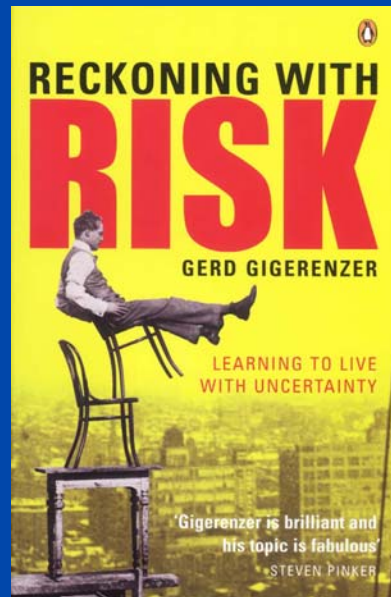
Mental calculators that perceive risk and how we may educate them

Kurz-Milcke, Elke, and G. Gigerenzer

Abstract: In this talk we address the costs and spread of innumeracy, and most importantly, how to design a cure. Innumeracy is not a mental disease, but largely caused by the representation of information (aside from conflicts of interest). Consequently, we propose a “cure” designed around transparent forms of external representation, most prominently around the use of natural frequencies. Generally, experts and laypersons should know and use, but also demand these transparent forms of representation in risk communication. The mental model implicated by the use of natural frequencies takes the form of a tree-like structure. We elaborate the features and possibilities of this particular mental model, and its place in the teaching of statistics from elementary school to professional training and continuing education.

**Calculated Risk:
How to know when
numbers deceive you.**

Simon & Schuster



Mental calculators that perceive risk and how we may educate them

- innumeracy
- the “cure”: transparent representations
- prevention: statistical literacy
- “Lebensgefühl”

Risk Communication

Representations that produce innumeracy

Single-event probabilities

Conditional probabilities

Relative risks

Risk Communication

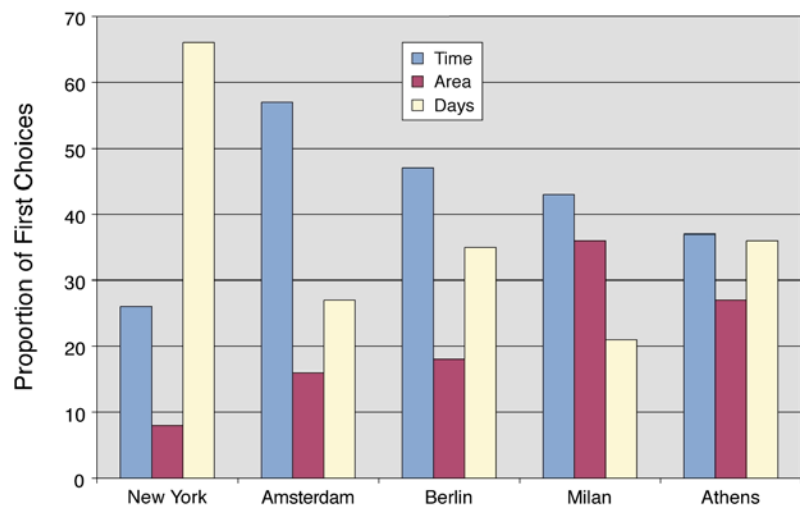
Representations that produce innumeracy

Single-event probabilities

Conditional probabilities

Relative risks

"There is a 30% chance of rain tomorrow"



Gigerenzer, Hertwig, van den Broek, Fasolo, & Katsikopoulos, *Risk Analysis* (in press)

Prozac's Side Effects

"You have a 30 to 50 percent chance of developing a sexual problem"

"Out of every 10 of my patients to whom I prescribe Prozac, 3 to 5 experience a sexual problem."

"Something goes awry in 30% to 50% of my sexual encounters!"

DNA Fingerprinting

"The probability that this match has occurred by chance is 1 in 100,000."

"Out of every 100,000 people, 1 will show a match."

Risk Communication

A conceptual framework for overcoming innumeracy

Reference class

Perspective taking

Natural frequencies

Risk Communication

Representations that produce innumeracy

Single-event probabilities

Conditional probabilities

Relative risks



What does a positive test mean?

A positive result means antibodies to HIV were found in your blood. This means you have HIV infection. You are infected for life and can spread HIV to others.

Illinois Department of
Public Health

Twenty AIDS Counseling Centers

Client: "If one is not infected with HIV, is it possible to have a positive test result?"

Counselor:

- | | |
|------------------------------------|---|
| 1 "No, certainly not" | 11 "False positives never happen" |
| 2 "Absolutely impossible" | 12 "With absolute certainty, no" |
| 3 "With absolute certainty, no" | 13 "With absolute certainty, no" |
| 4 "No, absolutely not" | 14 "Definitely not" ... "extremely rare" |
| 5 "Never" | 15 "Absolutely not" ... "99.7% specificity" |
| 6 "Absolutely impossible" | 16 "Absolutely not" ... "99.9% specificity" |
| 7 "Absolutely impossible" | 17 "More than 99% specificity" |
| 8 "With absolute certainty, no" | 18 "More than 99.9% specificity" |
| 9 "The test is absolutely certain" | 19 "99.9% specificity" |
| 10 "No, only in France, not here" | 20 "No comment" |

Gigerenzer, Hoffrage & Ebert, 1998, *AIDS CARE*

Innumeracy: An AIDS Counseling Session

Client: If one is infected, is it possible to receive a negative test result?

Physician: When there are enough antibodies, the test identifies them. If you are infected, you get a positive result with a 99.9% chance.

Client: And if I am not infected, can I get a positive result?

Physician: If you take the test here, including a confirmatory test, the test is extremely certain: in any case, the specificity is 99.9%.

Client: How frequent is the virus in my risk group, that is, among heterosexual men with no known risk factor such as IV drug use?

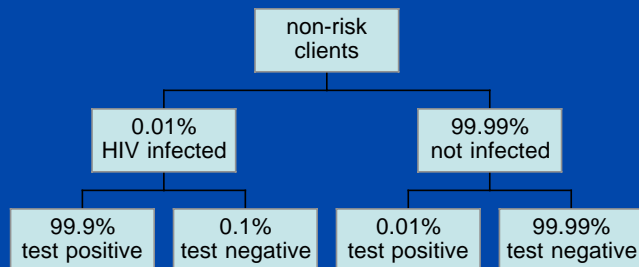
Physician: 1 in 1,000 or less.

Client: What are the chances that men in my risk group actually have HIV given a positive result?

Physician: As I already have said: extremely certain, 99.9%.

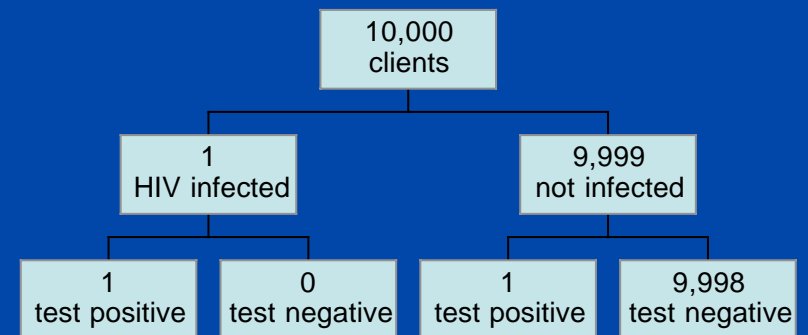
Gigerenzer, Hoffrage & Ebert, 1998, *AIDS CARE*

What is the chance of an HIV infection given a positive screening test?

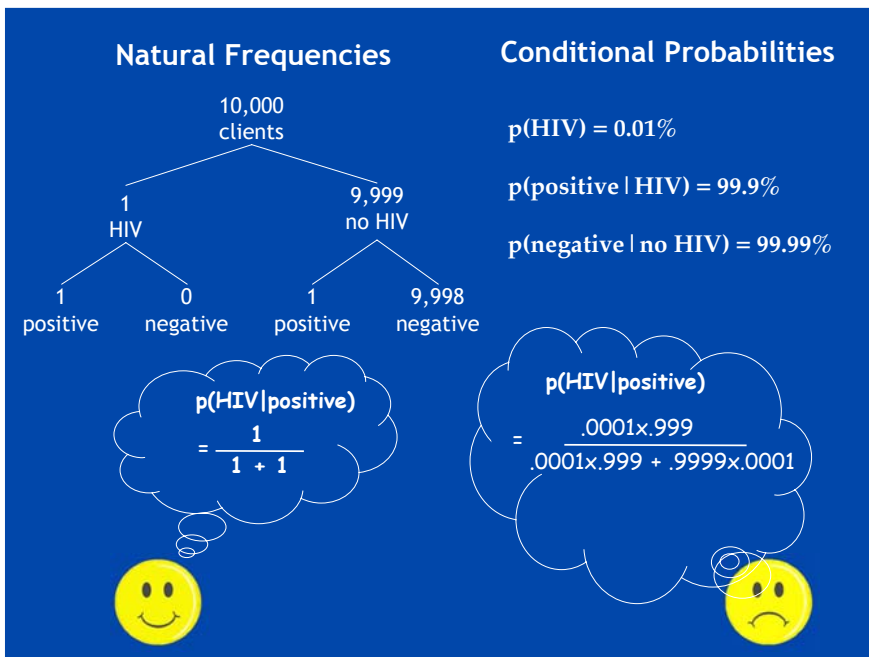


Representation: Conditional probabilities or relative frequencies

What is the chance of an HIV infection given a positive screening test?



Representation: Natural frequencies



Risk Communication

A framework for overcoming innumeracy

Reference class *Perspective taking*

Natural frequencies

Risk Communication

Representations that produce innumeracy

Single-event probabilities *Conditional probabilities*

Relative risks

How to raise anxiety

Official Statement: **Relative Risk**

The contraceptive pill* is associated with a 100% increase in the risk of thromboembolism.

What it means: **Absolute Risk**

The risk of thromboembolism increases from about 1 to 2 in 14,000 women.

*Combined oral contraceptives containing desogestrel and gestodene. Jain, McQuay & Moore (1998)

Benefits of Mammography Screening

Deaths from breast cancer (per 1000 women)

No screening	4
Screening	3

Three ways of presenting the benefits:

Relative risk reduction: Mammography screening reduces the risk of dying from breast cancer by 25 percent

Absolute risk reduction: Mammography screening reduces the number of women who die of breast cancer by 1 out of 1,000—that is, 0.1 percent

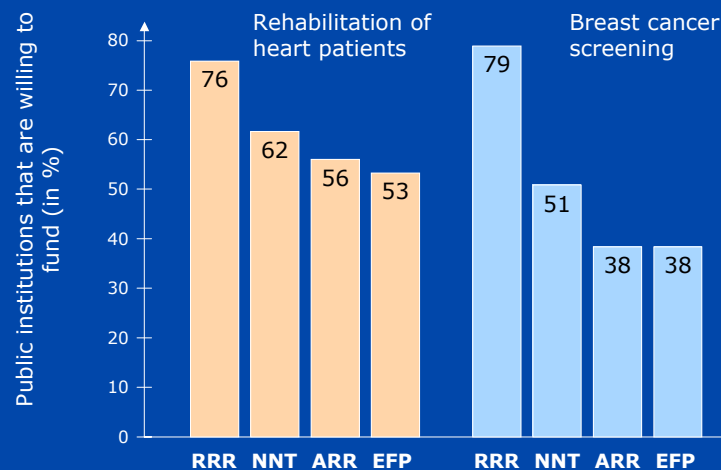
Number needed to screen: In order to prevent one death, the number of women who need to participate in screening for 10 years is 1,000

What information do women get?

Information	% of leaflets	
	Australia 1997	Germany 2001
Benefit		
Relative risk reduction	22	7
Absolute risk reduction	NEVER	7
NNT (number needed to screen)	NEVER	4
Harm		
Consequences of false positives	14	11
Discovery & treatment of non-progressive cancers	NEVER	11
Cancer caused by mammography	NEVER	44
Quality		
Sensitivity (or false negative rate)	26	19
Specificity (or false positive rate)	NEVER	4
Positive predictive value	NEVER	15

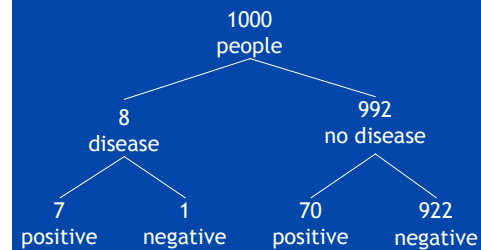
GERMANY: 27 leaflets on mammography screening by Bundesamt für Strahlenschutz, Deutsche Krebsgesellschaft, Krebsinformationsdienst, Krankenkassen, Mammographie-Screening Planungsstelle u. a. Source: Kurzenhäuser (2003). AUSTRALIA: 58 leaflets, Slaytor & Ward (1998), *British Medical Journal*

How to Get Funding



Fahey, Griffiths & Peters (1995), *British Medical Journal*

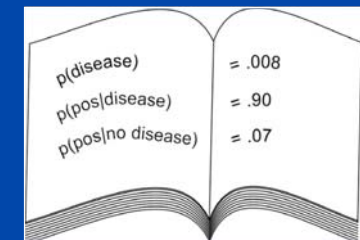
Natural Frequencies



$$p(\text{disease}|\text{positive}) = \frac{7}{7 + 70}$$



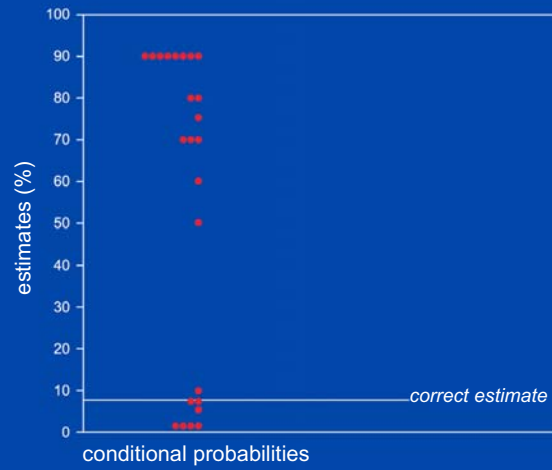
Conditional Probabilities



$$p(\text{disease}|\text{positive}) = \frac{.008 \times .90}{.008 \times .90 + .992 \times .07}$$

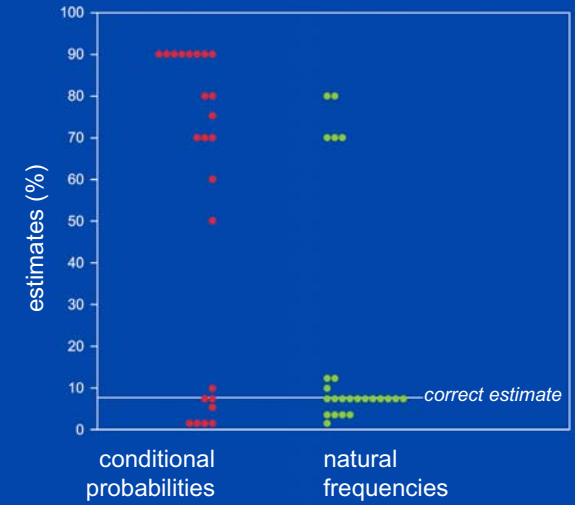


Mammography



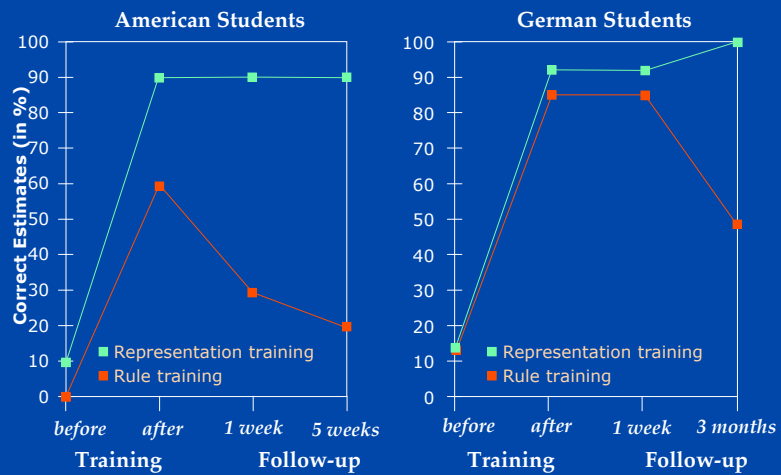
Gigerenzer (2002), *Reckoning with risk*. Penguin. (US version: *Calculated risks*. Simon & Schuster.)

Mammography

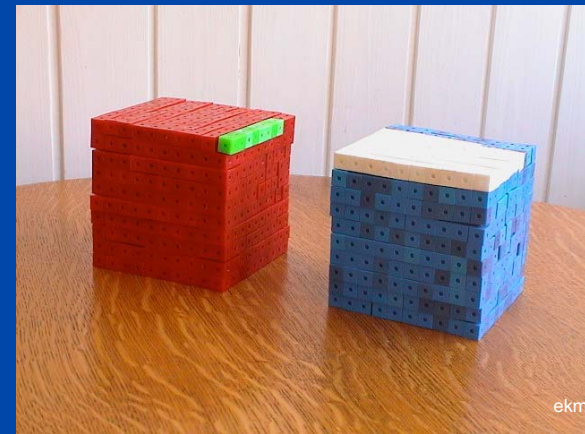


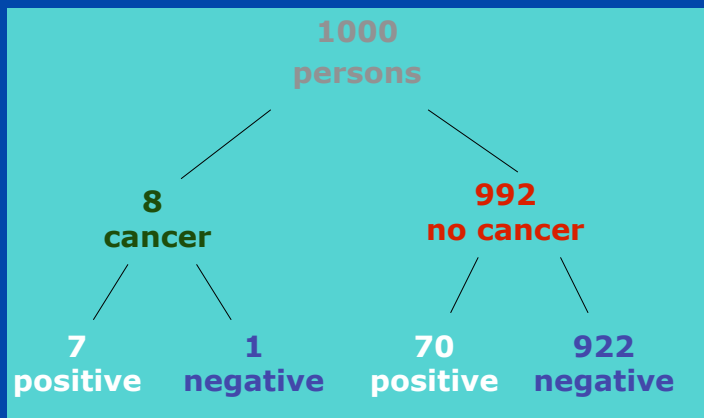
Gigerenzer (2002), *Reckoning with risk*. Penguin. (US version: *Calculated risks*. Simon & Schuster.)

Teaching Bayesian Reasoning in Less Than Two Hours



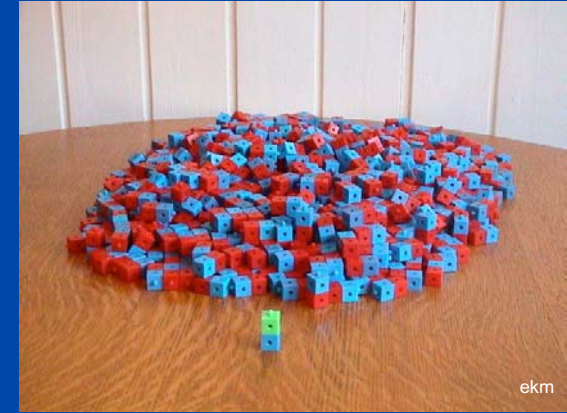
Sedlmeier & Gigerenzer (2001), *Journal of Experimental Psychology: General*





sorting, comparing, counting





	positive	negative	
cancer	7	1	8
no cancer	70	922	992
	77	923	1000

Fourth Graders



Kurz-Milcke, E. & Martignon, L. (2006). Lebendige Urnen und ereignisreiche Bäume. In: *Arbeitsbericht des AK 'Stochastik in der Schule'*, Gesellschaft für Didaktik der Mathematik. Hildesheim: Franzbecker.



"We-Urn"



12 nametags
13 nametags

17 girls
8 boys



The boys and the girls



Narrating urns

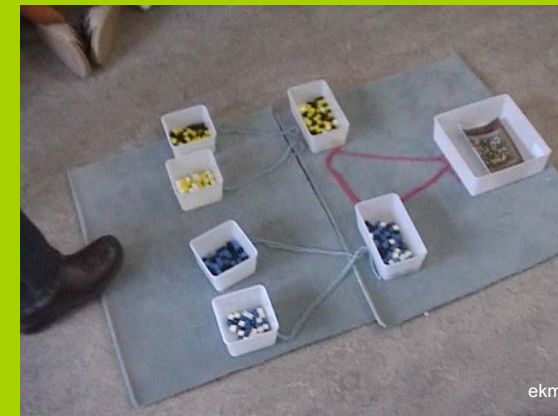
Sports arena

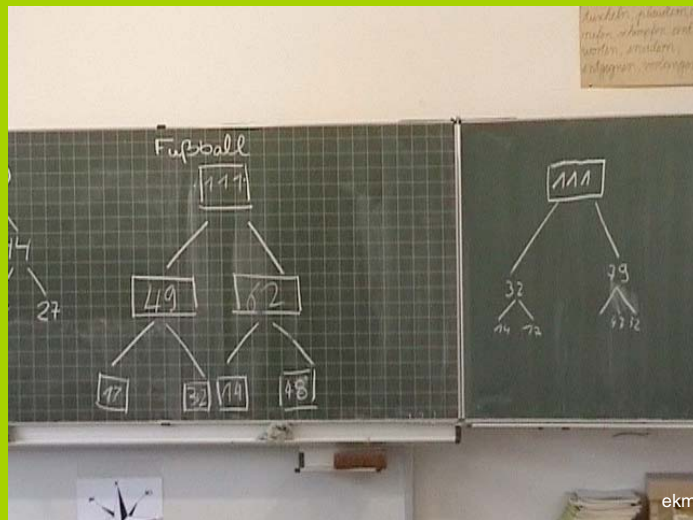
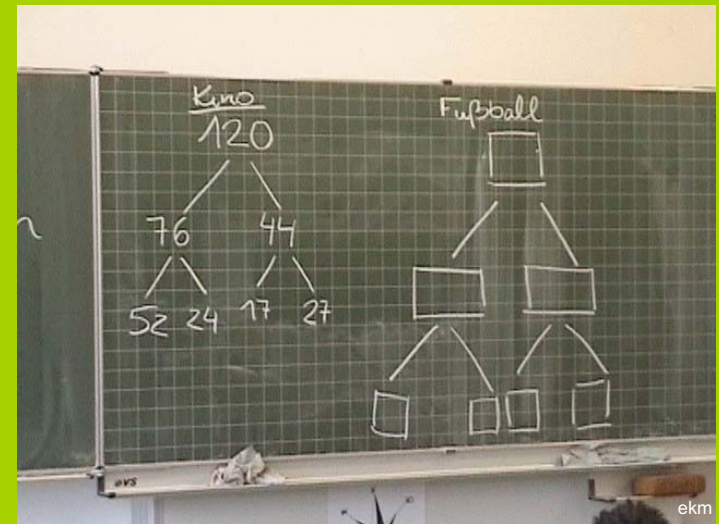
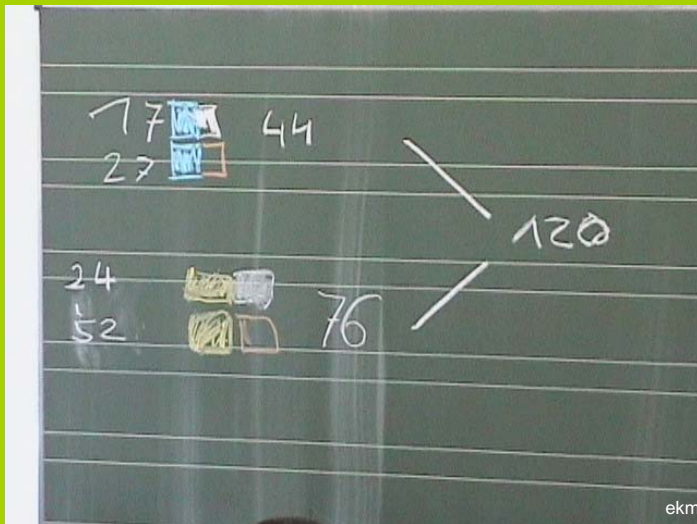


Movie theatre



Lively urns and eventful trees





A game around the design of binary trees



As presented in Engel, A., Varga, & Walser (1974).

Composing equivalent "similar" urns



ekm

ekm

What's in a recipe, i.e., urn?

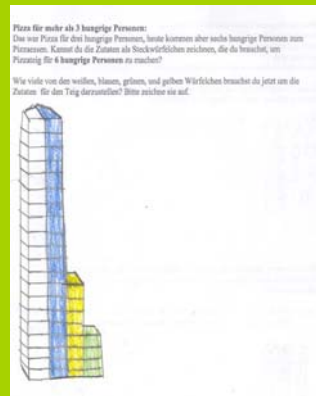


ekm



ekm

Representations by 4th-Graders



"Similar" urns



ekm



ekm



ekm



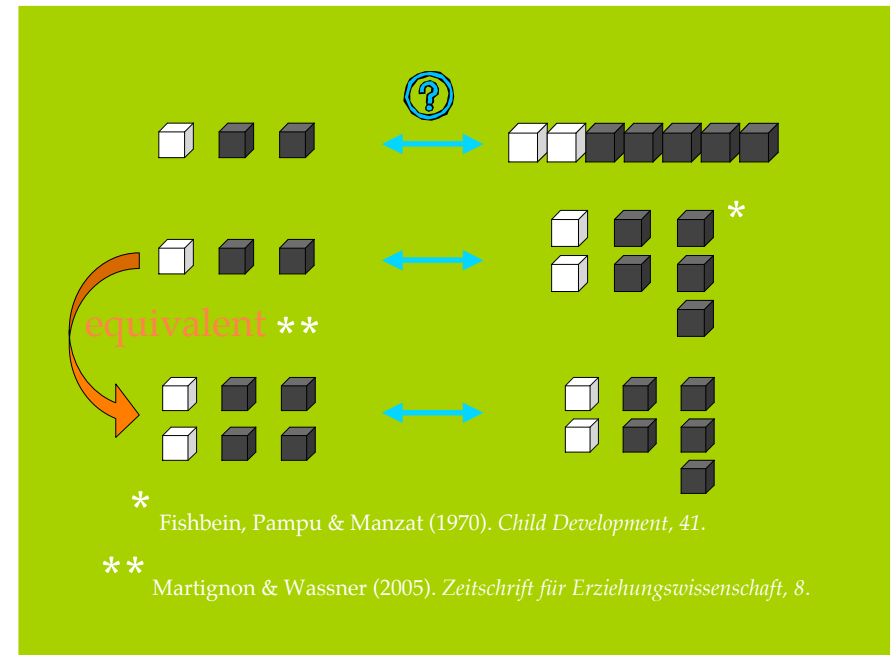
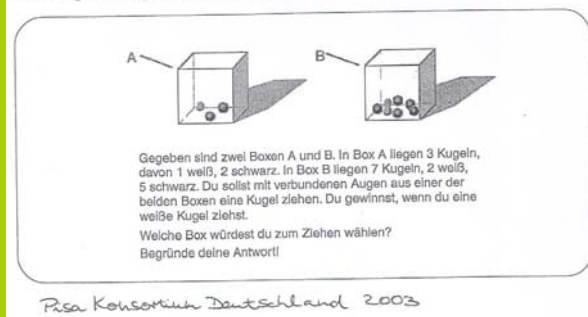
ekm

Martignon, L. & Kurz-Milcke, E. (2006). Bunte Steckwürfel und Kärtchen in Haufen: Wege zu einer natürlichen, geschlechter-sensitiven Stochastik in der Grundschule. In: *Arbeitsbericht des AK, Stochastik in der Schule*, Gesellschaft für Didaktik der Mathematik. Hildesheim: Franzbecker.

Comparing urns

Martignon, L. & Wassner, C. (2005). Schulung frühen stochastischen Denkens von Kindern. *Zeitschrift für Erziehungswissenschaft*, 8, 202-222.

Abbildung 2.9: Aufgabenbeispiel „Kugelbox“



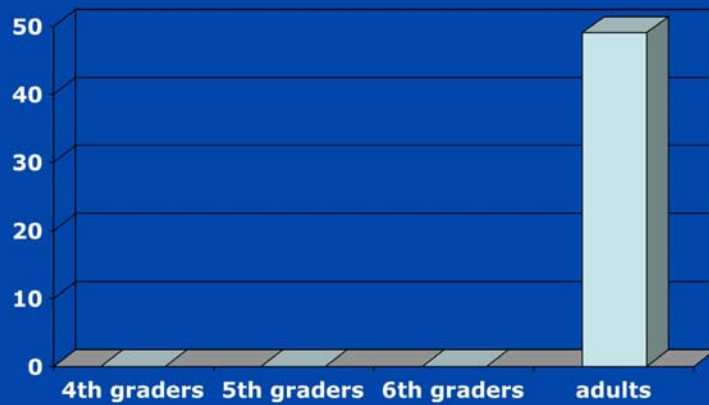
Red Nose Problem

Conditional Probabilities

Pingping goes to a small village to ask for directions. In this village, the probability that the person he meets will lie is 10%. If a person lies, the probability that he/she has a red nose is 80%. If a person doesn't lie, the probability that he/she also has a red nose is 10%. Imagine that Pingping meets someone in the village with a red nose. What is the probability that the person will lie?

Zhu & Gigerenzer, *Cognition*, 2006.

Bayesian Reasoning With Conditional Probabilities



Zhu & Gigerenzer, *Cognition*, 2006.

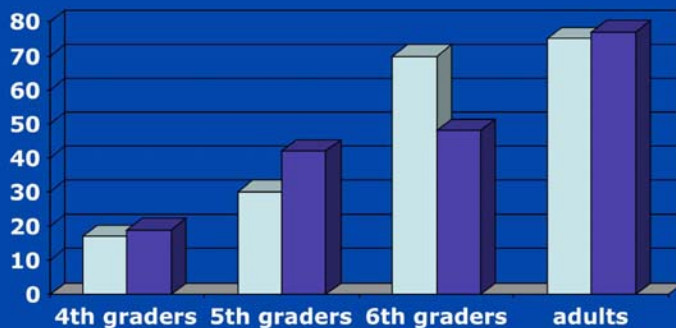
Conditional probabilities

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Natural frequencies

Pingping goes to a small village to ask for directions. In this village, 10 out of every 100 people will lie. Of the 10 people who lie, 8 have a red nose. Of the remaining 90 people who don't lie, 9 also have a red nose. Imagine that Pingping meets a group of people in the village with red noses. How many of these people will lie? ___ out of ___.

Bayesian Reasoning With Natural Frequencies



Study 1 (n=68, 7 problems)
Study 2 (n=120, 10 problems)

Zhu & Gigerenzer, *Cognition*, 2006.

Confusing natural frequencies with any kind of frequencies

- "data in the form of frequencies by no means guarantee good Bayesian reasoning" (Johnson-Laird et al., 1999, *Psych. Rev.*). J-L presented an experiment with normalized, not natural frequencies.

- "we are not convinced that it is frequency information per se which is responsible for the facilitation" (Evans et al., 2000, *Cognition*). Normalized frequencies showed no facilitation; therefore the authors rejected the "frequentist hypothesis." The same confusion in Girotto & Gonzales (2001, *Cognition*), Lewis & Keren (1999, *Psych. Rev.*), and Macchi (2000, *OBHDP*).

- The "point made by Gigerenzer is that the use of frequency reliably makes cognitive illusions 'disappear' ... this statement is just wrong" (Kahneman & Tversky, 1996, *Psych. Rev.*).

- Gigerenzer's "'frequencies, good; probabilities, bad' critique ... simply does not hold up empirically. In fact, presenting frequencies rather than probabilities sometimes makes judgment distinctively worse ..., sometimes makes judgments distinctively better" (Gilovich & Griffin, 2002).

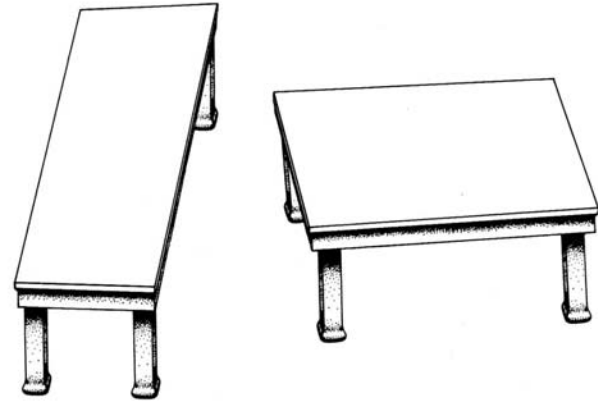
The Illusion of Certainty

"... in this world there is nothing certain but death and taxes."

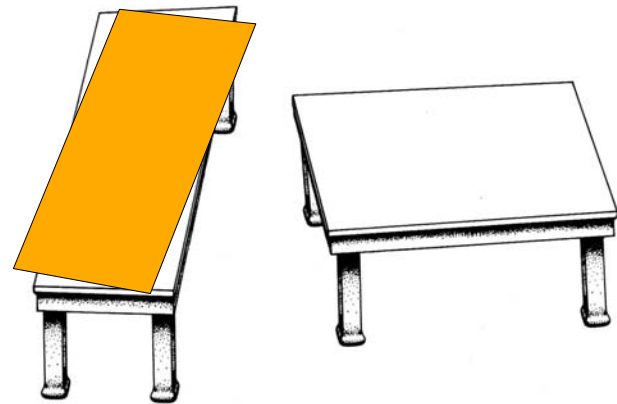
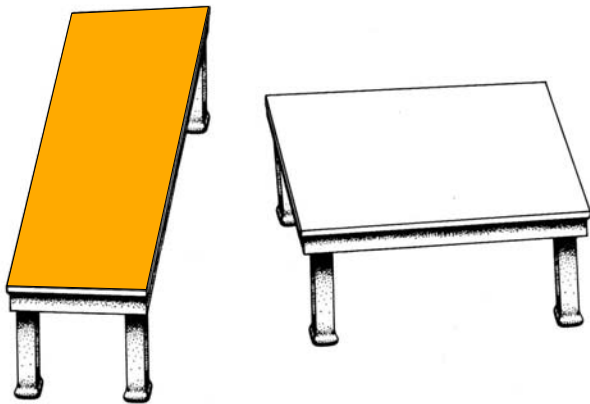
Benjamin Franklin

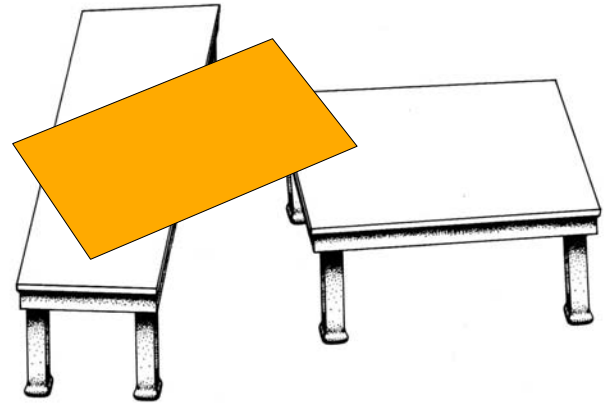
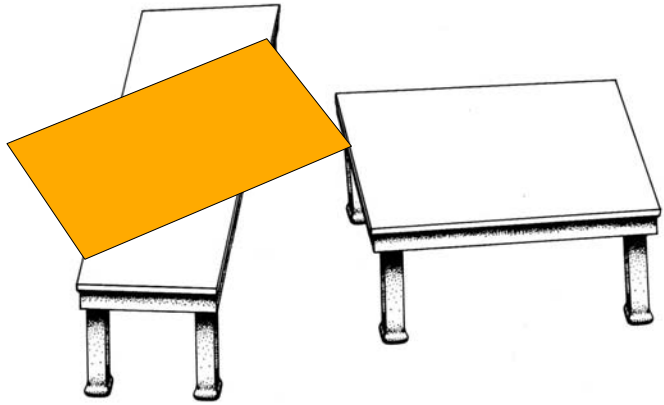
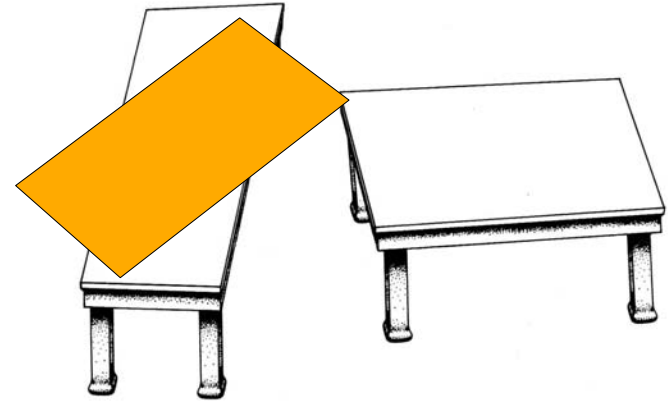
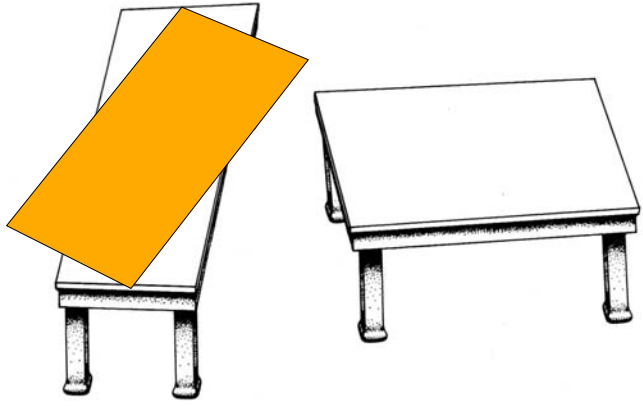
"... if I were to tell my patients what we do not know, they would get very, very nervous."

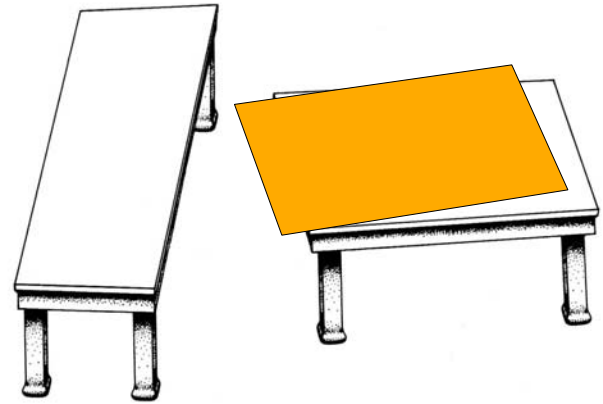
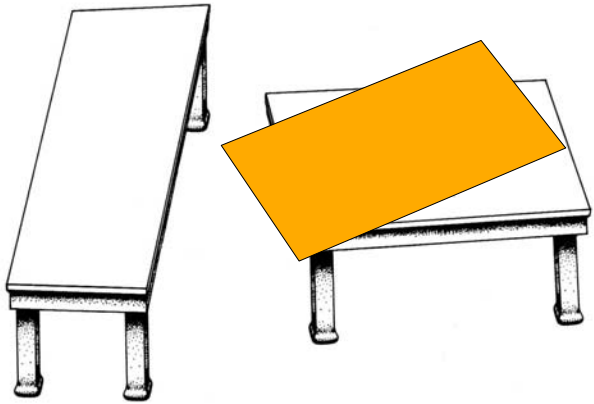
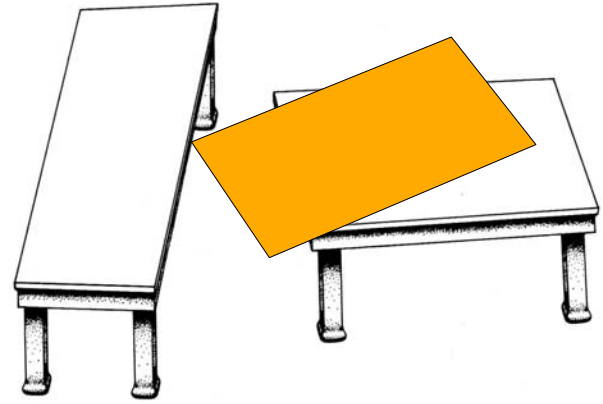
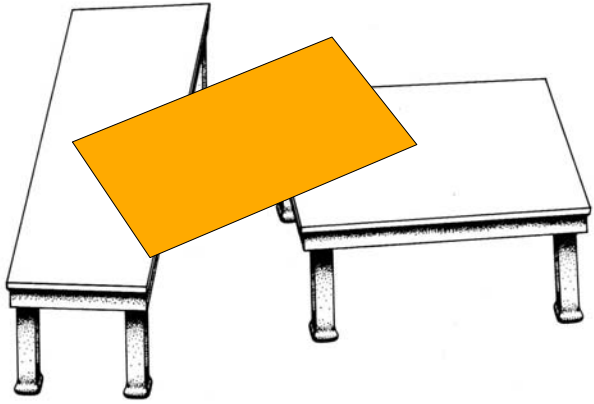
A surgeon

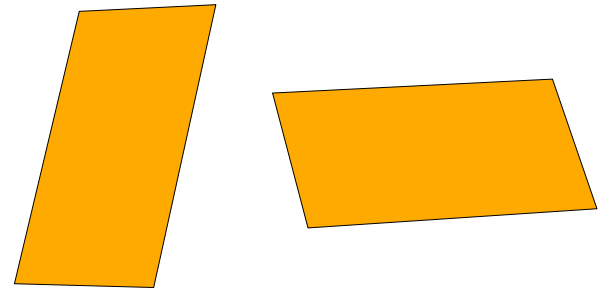
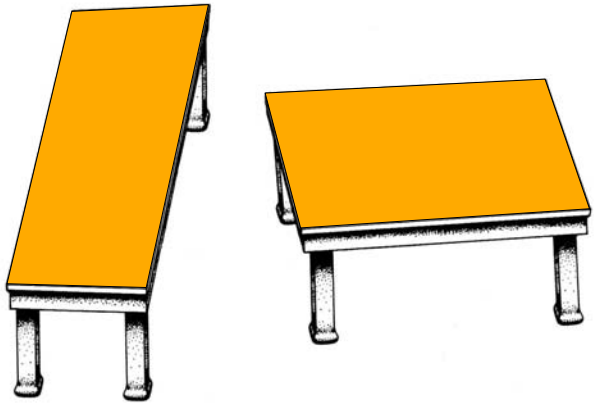
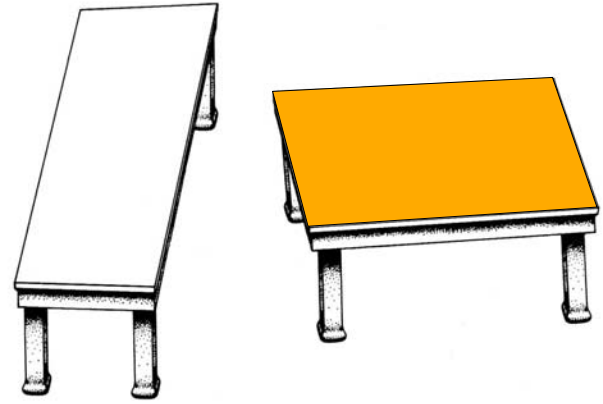
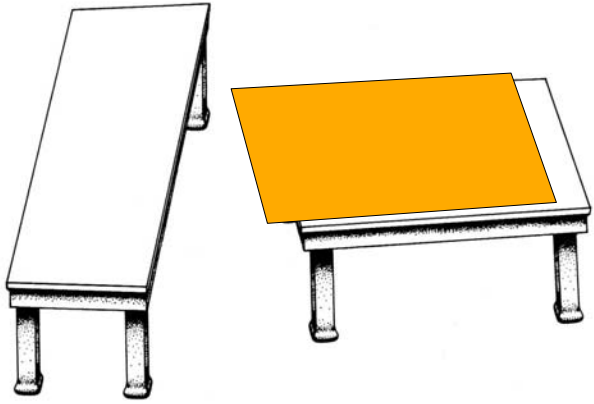


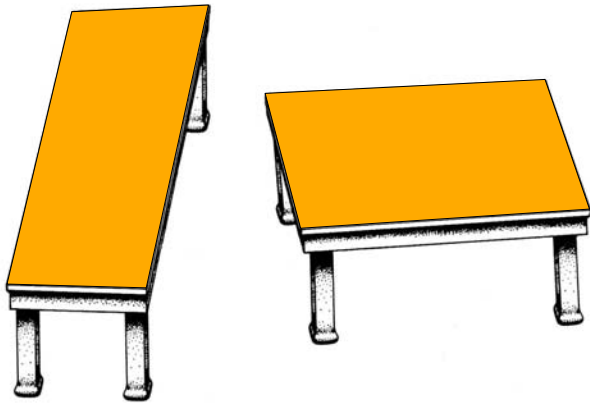
Shepard (1990)









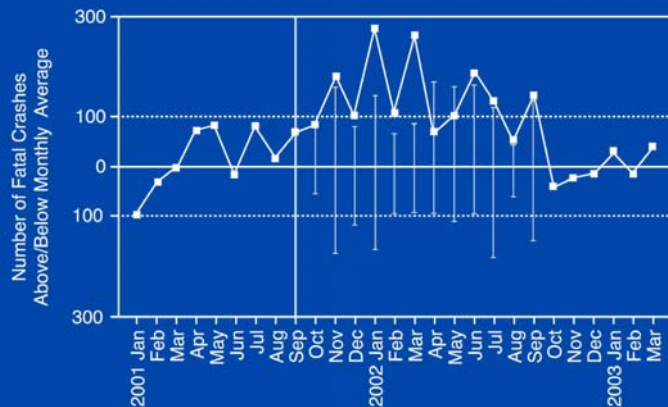


Is the illusion of certainty simply a matter of knowing?

"After 9/11, I explained the greater risk of driving compared to flying to my wife; that did not do the trick."

A Professor of Psychology, Loyola University, USA

Fatal Crashes



Funding for HIV/AIDS:

Cumulatively through the fiscal year 2004, the US government has invested approximately \$150 billion for domestic and international HIV/AIDS programs.

For 2005, the President's budget request includes \$19.8 billions, a proposed increase of 7%.

Elke Kurz-Milcke is a cognitive psychologist working in the Institute of Mathematics and Computing at the Pedagogical University (Hochschulleitung und Studienort Ludwigsburg) of Ludwigsburg, Germany. She holds a Ph.D. from Bowling Green State University in Ohio. She has been a researcher at the Center for Adaptive Behavior and Cognition at the Max Planck Institute in Berlin and at the College of Computing at the Georgia Institute of Technology in Atlanta. Her specialty is in bridging the cognitive study of science and of education. She is contributing to a state-sponsored research effort on gender and mathematics. Her current research addresses, among other issues, how to best introduce statistics and probability to children. While a graduate student and a postdoctoral fellow, she earned various prizes and honors, among them a Schloßmann-Fellowship from the Max Planck Society for the Advancement of Science. She has published on representational practices in mathematics, on research practices and modeling in biomedical engineering, on the history of research in psychology and in decision making, as well as on expertise.